

The Washington ABE
Use Math to Solve Problems
and Communicate
Curriculum Framework



Adult Basic Education
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The Washington Curriculum Framework for Use Math to Solve Problems and Communicate

This document is the Washington state curriculum framework in ABE math. This curriculum framework attempts to accurately represent Washington state's teaching and learning objectives, as well as viable strategies, knowledge, and skill sets to reach them.

Equipped for the Future's (EFF) curriculum framework, together with Washington state adult educators' professional wisdom and classroom experiences, have provided a rich matrix of ways to make these teaching and learning objectives (TLOs) real. The TLOs themselves, the recommended strategies, skills and knowledge to reach those objectives, the examples of proficient performance, examples of Persons Performing Math to Solve Problems, and examples of Teaching and Learning Math are valuable resources. All of these descriptive elements are available to us as a field for investigation, review and further adaptation. The WA Learning Standards Cadre will have a key role in this investigation, as do the staff of SBCTC-ABE, and teachers and learners throughout the state.

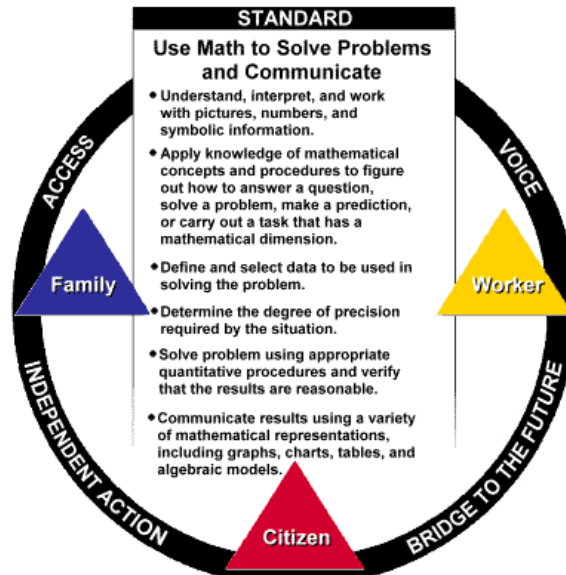
Please use and distribute this document widely. Also, please pay attention to and collect your own reactions to its validity. We look forward to a shared dialog in creating the Washington State Adult Learning Standards' curriculum frameworks.

Introduction

A curriculum framework is a guide to teaching and learning which outlines the general content and goals of instruction at different levels. The EFF *Use Math to Solve Problems and Communicate* Curriculum Framework is based on the EFF Standard *Use Math to Solve Problems and Communicate* and gives teachers tools to use in the instruction and assessment of adults' numeracy. This framework is not a math curriculum but provides a basic structure which teachers can use to design a curriculum that is relevant to the needs of their particular group of students.

The EFF Standard *Use Math to Solve Problems and Communicate*

In order to fulfill responsibilities as parents/family members, citizens/community members and workers, adults need to be able to:



The EFF Standard *Use Math to Solve Problems and Communicate* describes the use of math as a process of applying knowledge, skills and strategies to solve problems and communicate. It describes the application of math proficiency to accomplish adult tasks in the real world.

The ability to perform tasks having a mathematical dimension is the focus of the standard. The first part of the standard states that the individual must understand, interpret, and work with pictures, numbers, and symbolic information; however, this is only one aspect of the standard. Working with numbers and doing computations serves a purpose: to solve a problem or to communicate. Choosing the data needed and determining the degree of precision (how accurate is accurate enough) depends on the situation and the purpose. An individual does not necessarily carry out the components of the standard in sequential, discrete steps. Rather, the components are integrated as the individual draws on sets of underlying skills in order to accomplish his or her purposes for using math. The same is true for math instruction; it should always begin with a real purpose for having to work with pictures, numbers and symbolic information. But instruction and learning may occur in many combinations and sequences and will, of course, vary as skill changes.

How the EFF *Use Math to Solve Problems and Communicate* (UMSPC) Curriculum Framework Is Organized

The UMSPC Curriculum Framework was developed in reference to the Performance Indicators¹ of the UMSPC Performance Continuum. The **Teaching & Learning Objectives** for each level were determined by keeping the performance indicators of that level clearly in focus and answering the question, “What kinds of knowledge, skills, and strategies support developing math users in achieving these performance indicators?”

Each of the five performance levels is presented in a consistent format. On the first pages of the level you will find:

- **The defining components of the *Use Math to Solve Problems and Communicate* standard.**
- **Performance Indicators for the Specific Level.** Knowledge, skills, and strategies which learners at this level draw from in life-based math.

Together, these address the key features, or dimensions, of performance at any level:

- structure and depth of the knowledge base;
- fluency of performance;
- independence of performance;
- range of performance.

- **Teaching and Learning Objectives.** The objectives are organized in four strands:
 - A. Number Sense
 - B. Patterns, Functions, Relationships
 - C. Space, shape, measurement
 - D. Data and Statistics

These strands give guidance for teaching and learning at this level, with the specific knowledge, skills, and strategies most likely to be useful at the particular level.

Each strand is divided into objectives, notated by the upper-case letter followed by a number (e.g., **A1, A2**, etc.). The objectives have been constructed so that each notated objective addresses the same content in every level. For example, A3 addresses determining the degree of precision in each of Levels 1 through 5; B1 addresses reading, writing, and interpreting patterns, functions, and relationships at each of the levels. The objectives have been designed in this fashion to facilitate ease of reference.

Under many objectives you may find items notated by lowercase letters. These present specific knowledge content or strategies that might be helpful to teach at this level. When knowledge or strategies are mentioned in a lower level and not in higher level, it may be assumed that they still

¹ NOTE: These performance indicators depict behaviors that are expected at the exit point of a level. They are designed to inform the development of accountability assessments that document movement from one level to the next. As such, they are integral to the development of guides to instruction.

apply. If a teacher observes that a student is not demonstrating the particular knowledge, skill, or strategy listed in a lower level--and would benefit from developing it—that content should be added to instruction.

- **Examples of Proficient Performance at this Level.** These are examples of **meaningful real-life tasks** in which adults use the standard at this level to accomplish important purposes in their roles as citizens, family members and workers.

Following the first two pages of each level are pages which provide some context for the Performance Indicators and Teaching and Learning Objectives.

- **Examples of a Person Performing at this Level.** These present examples of how an individual proficient at this level integrates sets of knowledge, skills, and strategies to accomplish a task involving a mathematical dimension. These examples articulate our goal for teaching and learning – to support development of the knowledge and skills necessary for independent and fluent performance of the standard at this level in important, real-life adult activities.
- **Examples of Teaching and Learning at this Level.** These are descriptions of the kinds of teaching and learning activities that might support an individual in becoming proficient in using math at this level. You will see that the instruction described here is designed to target and integrate specific knowledge, skills and strategies that support performance at that level articulated as Teaching and Learning Objectives.

Using the Curriculum Framework

1. How will this Curriculum Framework help me know what to teach?

The first question many teachers ask when they examine this curriculum framework is “How do I begin to locate my students in the levels of performance so that I will know where to begin the teaching and learning process?”

EFF does not include a diagnostic assessment, but the **Performance Indicators** will help to answer this question. They describe what a student can accomplish in general terms and some specific examples of how a student might carry out tasks at each level of proficient performance.

The **Guide to Using the Use Math to Solve Problems and Communicate Performance Continuum²** provides the following suggestion to using the Performance Continuum to assess prior knowledge:

1. Look over the Performance Continuum to familiarize yourself with what is required for proficient performance at each level.
2. Review what you already know about your students, based on intake assessments you or your program may have conducted, standardized test data, student portfolios, and previous student work. Determine the performance levels that appear to best reflect your students' proficiency levels.
3. Gather additional information you need by asking students to demonstrate and/or reflect on what they already know. Step 2 of the Teaching and Learning Toolkit³ has several tools you can use. Tailor the questions you ask to fine-tune your understanding of the performance levels of your students.

After determining the performance levels of students, the teacher may then turn to the Teaching and Learning Objectives to design instruction based upon her own and program goals. The Curriculum Framework outlines specific instructional objectives at each level that can be integrated into purposeful learning based on student goals. The activities and strategies in each strand help teachers determine what to teach to meet the needs of specific students and help students master the *Use Math to Solve Problems and Communicate* Standard at each level.

2. How will I know that my students have learned from my instruction, using the Curriculum Framework?

As you plan instruction, you will also be planning how you will collect evidence of what students have learned. One way is by the use of observation and documentation in a journal. Student progress can also be monitored in a well-defined portfolio, a systematic collection of work, created over time, which may include entries such as representative products, the best work of the student, or indicators of progress. (The EFF Teaching and Learning Toolkit has additional ideas on how to collect evidence.) Then use the Performance Indicators and the Examples of Proficient Performance to evaluate this evidence.

²<http://eff.cls.utk.edu/assessment/math1.htm>

³<http://eff.cls.utk.edu/tools.htm>

Levels 1 & 2: Use Math to Solve Problems and Communicate

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models.

NOTE: WA Levels 1 & 2 are currently both addressed by EFF Level 1 TLOs. Level specific TLOs to be developed with WA LS Cadre.

WA Level 1 Indicators

M 1.1 Read, write, and interpret very simple types of mathematical information such as

Numbers and number sense: whole numbers (three digit numbers), common monetary values, and benchmark fractions ($1/2$, $1/4$).

Patterns/Functions/Relationships: very simple patterns, commonly-used denominations/groupings (2s, 5s, 10s); mathematical relationships – more, less, larger, smaller, left, right, heavier, longer.

Space/Shape/Measurement: high frequency standard units of measurement (pounds, feet, months, weeks, days, minutes, hours), and concepts of geometric shape, length and width.

Data/Statistics: very simple ways to interpret and represent data (checksheets, picture graphs,) emphasizing frequency of occurrence.

M 1.2 Recall and use a few simple mathematical procedures such as very basic estimating, counting, sorting, ordering, grouping, adding on (using counting or a calculator), orally counting by 2s, 5s and 10s, addition and subtraction and beginning multiplication.

M 1.3 Evaluate the degree of precision needed for the solution.

M 1.4 Extract discrete information from simple and concrete data and graphs, describe patterns, and/or use basic computational procedures effectively to solve a problem and to verify that the solution is reasonable.

M 1.5 Communicate the solution to the problem orally, in role plays, with pictures, or by entries on a simple chart.

SHOW FLUENCY, INDEPENDENCE, AND ABILITY TO PERFORM IN A RANGE OF SETTINGS

Adults performing at Level 1 can select and apply the knowledge, skills, and strategies at this level to independently, with some effort but few errors, accomplish simple, well-defined, and highly structured math tasks in one or more comfortable and familiar settings.

WA Level 2 Indicators

M 2.1 Read, write, and interpret very simple types of mathematical information such as

Numbers and number sense: whole numbers (three digit numbers), common monetary values, and benchmark fractions ($1/2$, $1/4$) and percents (50%).

Patterns/Functions/Relationships: very simple patterns, commonly-used denominations/groupings (5s, 10s, 25s), and very simple proportions (2:1, 1:2).

Space/Shape/Measurement: high frequency standard units of measurement (pounds, feet, quarts, gallons), geometric shapes, and concepts of length and width.

Data/Statistics: very simple ways to interpret and represent data (checksheets, picture graphs, unambiguous bar graphs, line plots) emphasizing frequency of occurrence.

M 2.2 Begin to evaluate reasonableness of solutions. Add and subtract whole numbers through three digits, and multiply and divide three digit numbers by one digit numbers. Recall and use mathematical procedures such as basic estimating, counting, sorting, ordering, grouping, adding on (using counting or a calculator), and measuring length and weight using tools calibrated with whole numbers (rulers, manipulatives).

M 2.3 Evaluate the degree of precision needed for the solution.

M 2.4 Extract discrete information from simple and concrete data and graphs, and measure with appropriate tools, describe patterns, and/or use computational procedures effectively to solve a problem and to verify that the solution is reasonable.

M 2.5 Communicate the solution to the problem orally, in role plays, with pictures, or by entries on a simple chart.

SHOW FLUENCY, INDEPENDENCE, AND ABILITY TO PERFORM IN A RANGE OF SETTINGS

Adults performing at Level 2 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish simple, well-defined, and highly structured math tasks in one or more comfortable and familiar settings.

Teaching and Learning Objectives

To support performance of the Standard at Level 1 & 2, instruction should be offered in the context of real-life purposes, texts, and activities, and should, across the four strands, address the integration of knowledge, skills, and strategies such as those below:

Number Sense Strand

1&2A1 Read, write, and interpret very simple types of mathematical information:

- Read, write, and compare whole numbers up to three digits
- Understand and use place value to hundreds
- Count by 2s, 5s, 10s
- Recognize common monetary values
- Demonstrate an understanding that 50% is the same as $\frac{1}{2}$ and 25% is the same as $\frac{1}{4}$

1&2A2 Recall and use a few simple procedures:

- Estimate with numbers up to 3 digits
- Sort, order, group by color, shape, etc.
- Add-on using counting or calculator
- Subtract, including the understanding that addition and subtraction are related
- Demonstrate an understanding of the multiplication facts up to 12 (such as repeated addition, a combination of multiplication and adding on, etc.)
- Demonstrate an understanding that add and multiply mean to combine, subtract means to take away, and divide means to separate into groups

1&2A3 Determine degree of precision, rounding to the nearest 10, 100

1&2A4 Use computational procedures effectively to solve real-life problems:

- Accurately adds and subtracts one and two digit numbers using manipulatives, grouping, or calculator
- Determine one-half or one-quarter of an amount using manipulatives, drawings, or other simple strategies
- Determine 50% or 25% of an amount
- Can double very simple whole number amounts
- Can apply simple proportions such as 2:1, 1:2
- Use calculator and drawings to check for reasonableness of answer

1&2A5 Communicate results in role plays, with pictures, and by entries on a simple chart

Patterns, Functions, Relationships Strand

1&2B1 Read, write, and interpret very simple patterns, functions and relationships:

- Recognize and complete number patterns up to 1000 and generalize the relationship
- Create tables for addition and multiplication

Levels 1 & 2 Examples of Proficient Performance

Adults performing at Level 1 & 2 can count, order, and group to accomplish a variety of goals, such as:

- Make change using the "building" method
- Determine the amount of flooring needed in a room by counting square units of material within the shape of the room
- Estimate daily/weekly wages by adding on (counting) per-hour amounts
- Fit furniture into a space by counting square units within the shape of the space or by estimating using informal measurement units
- Interpret an unambiguous horizontal or vertical bar graph in a brochure from a local clinic in order to understand information about children's health concerns
- Measure ingredients for simple recipes using benchmark fractions
- Use tallies to determine number of useable items from total number of items produced
- Sort coins into like piles, and then determine the value of each pile
- Use manipulatives, mental math, a calculator, or paper and pencil to calculate how much it will cost for 2 people to go to the movies
- Develop a schedule for how and when to take medication according to a doctor's order
- Double a recipe for chocolate chip cookies for a children's party

1&2B2 Read and interpret symbolic information:

- a. Recognize and name all ten digits
- b. Recognize and name +, -, = symbols
- c. Can solve simple number sentences such as $3 + 2 = 5$
- d. Orally describe number relationships such as $3 + 2$ does not equal 7
- e. Read positive and negative numbers

Space/Shape/Measurement Strand**1&2C1 Read, write, and interpret very simple types of information related to measurement:**

- a. Read and write time (hours and minutes, day of week, etc.)
- b. Read measuring tools (rulers, measuring cups) to nearest whole unit
- c. Recognize and describe familiar geometric shapes (such as circle, square, triangle, etc.)
- d. Describe a shape based on dimensions of length and width
- e. Show equivalent amounts of money using different coins and dollar bills
- f. Use coordinate geometry to specify locations and relationships (such as east and west, north and south) on a map

1&2C2 Recall and use a few simple mathematical procedures:

- a. Measure involving high frequency standard whole units of measure such as pounds, inches, feet, quarts, gallons, cups
- b. Use appropriate tools to measure to the nearest whole unit
- c. Find the area and perimeter of rectangles using simple strategies such as drawings or adding on
- d. Calculate the total cost of several items and the amount of change due (for amounts up to \$20.00)

Data and Statistics Strand**1&2D1 Read and interpret data and statistical information:**

- a. Extract discrete information from simple and concrete graphs, or from a list or table
- b. Describe concrete information found in a graphic representation
- c. Make numerical comparisons about relative values on a bar graph (such as “greater than” or “less than” or “about twice as much”)
- d. Use titles and labels to make statements about data representations

1&2D2 Collect, organize, and represent data:

- a. Collect and organize data based on posed question and organize in a reasonable manner
- b. Represent data in simple forms such as check sheets, picture graphs, simple bar graphs, and line plots
- c. Use simple charts or tables to communicate information
- d. Verify that data represented is the actual data collected

1 & 2D3 Recall and use basic probability concepts:

- a. Describe the probability of an event within the range of “possible” or “impossible”
- b. Find the probability of a single outcome in a simple concrete situation such as a coin toss or die roll

Examples of a Person Performing at Levels 1 & 2:

Doubling Up

Tomasa is planning for her young son's birthday party. She wants to bake chewy graham bars. The basic recipe, which makes 20 bars, calls for the following ingredients:

3 eggs 1 1/2 cup graham cracker crumbs
3/4 brown sugar 1/2 teaspoon grated orange peel
1/2 teaspoon vanilla 1/8 teaspoon salt
1 cup fruit bits

But Tomasa wants more than 20 bars so she decides to double the recipe. Tomasa knows that, in order to double the recipe, she has to double all the ingredients. She has no problem doubling the number of eggs and the amount of fruit bits since she knows that two 3's are six and two 1's are two. Because she always struggled with fractions in school, she feels she cannot "do" fractions. She simply uses the measurements listed and measures each of the remaining ingredients twice. As she does so, she remembers that she does know that two halves make a whole, so she realizes that she actually uses 1 teaspoon of vanilla for her double batch.

She tells her son that there should be plenty of leftover bars for him to have for snacks later in the week because she knows that the double batch should make 40 bars. She has figured this out by adding 20 and 20 together.

Doing Time

Yolanda had to do community service for 30 hours a week. She needed to create a schedule so she would know when she should report for work. She also wanted to be able to show others that she was working the requisite number of hours.

Since she wanted to work her 30 hours during the weekdays to have her weekends free to spend with her children, she began by making five columns and writing in the initial letter for the five days of the week. She then thought about the time that her children are in school. The school bus picks them up about 7:15 each morning and drops them off at home at about 2:45 in the afternoon.

She figured she would prefer not to go to work until after the children leave for school and she would like to be home when they get home. She decided that she has from about 7:30 – 2:30 to deal with work.

She counted from 7:30 – 2:30 using tick marks to show her counting: 7:30 – 8:30 = 1; 8:30 – 9:30 = 1; etc. She had learned to group sets of five to make the counting easier. She figured that she has 7 hours total per day. She filled out her schedule like this:

M	T	W	T	F
7:30 – 2:30	7:30 – 2:30	7:30 – 2:30	7:30 – 2:30	7:30 – 2:30
II	II	II	II	II
7	7	7	7	7

She then added, then regrouped and added all her tick marks:

$$\begin{aligned} & \text{III II} + \text{III II} + \text{III II} + \text{III II} + \text{III II} \\ & \text{III} + \text{III} + \text{III} + \text{III} + \text{III} + \text{II} + \text{II} + \text{II} + \text{II} + \text{II} \end{aligned}$$

Since she knew how to count by 5s and by 2s, it was easy to count the total number of hours. She then used a calculator to check her answer.

She realized that the total was more hours than she needed to work, so she then decided to take an hour off for lunch each day. Her schedule now looked like this:

M	T	W	T	F
7:30 – 11:30	7:30 – 11:30	7:30 – 11:30	7:30 – 11:30	7:30 – 11:30
III	III	III	III	III
12:30 - 2:30	12:30 - 2:30	12:30 - 2:30	12:30 - 2:30	12:30 - 2:30
1	1	1	1	1
≠ 6 hours	≠ 6 hours	≠ 6 hours	≠ 6 hours	≠ 6 hours

She again checked her total number of hours and discovered that she had just enough hours. Now that she had a schedule that she could actually see, she knew that she could tweak it by moving hours and half-hours from one day to another, as long as she always gave the same number to one column as she took from another. She realized that she could give and take hours from one column (day) to another and still keep a total of 30 hours. She realized that, if she wanted to leave half an hour earlier, all she would have to do is shorten her lunch time by half an hour.

Examples of Teaching and Learning at Levels 1 & 2:

(Exit point for National Reporting System Beginning Basic Education)

“Halving” Fun

Gerald has a class of mostly women who have lost their jobs in a factory and are now working to upgrade their skills for service - related jobs. Some also want to get their GED. Almost all of the students are afraid of math and are clear that they do not “do” math. After much conversation, Gerald finds that he needs to focus their attention on math since it is one of the areas assessed on the GED and it is a needed skill for the workplace as well as for issues concerning the home and family.

The students agree that they need to focus on this area and they all mention how they are intimidated by fractions. Since “doing fractions” can mean performing operations with fractions (such as $\frac{1}{4} + \frac{1}{3}$ or $\frac{3}{4} \times \frac{4}{6}$) or it can mean using ratios in fraction form (such as $\frac{1}{3}$ of \$50). “Doing fractions” can also mean figuring out the points between whole numbers on rulers and such. Gerald first wants to find out what students already know about fractions. He also skims the GED Practice Tests to determine the types of fractions that are actually used in problems that appear on the GED Test.

Gerald asks the students how they use math in their lives now. He puts all their ideas on chart paper. Then he asks them in what situations they use fractions and the typical fractions they see. Through this conversation both Gerald and the students find out that they face fractions in many situations: from cooking (although they don’t tend to do much “operating” on the fractions) to shopping ($\frac{1}{2}$ off sale!) to talking about it in their lives (“You can take half of the cookies, but leave the other half for your sister.”)

Gerald provides several experiences in which students have opportunities to solidify their understanding of the ratio $\frac{1}{2}$, including its counterpart in percent (50%) and decimal format (0.5). They use chips to make the connection between half of a dollar and \$.50. They also look at various sets of items and determine what 50% is. They realize that 50% is always $\frac{1}{2}$ even though the amount that is in each half varies, depending on the total amount. This ‘aha’ is significant to understanding the concept of percent and to differentiating between relative vs. absolute amounts.

To see how well they can apply their understanding of $\frac{1}{2}$ to a real situation, Gerald and the students think about where they might hear about the idea of half. One of the students comments about hearing something on the news about “over half of the people voted against something”. They decide that they would like to do a survey to determine where students stand on various issues. Together they create a simple questionnaire with yes/no answers. Included on the questionnaire are items related to smoking (since this had been a topic of discussion during earlier lessons).

Students then tally the answers from their class as well as two other adult education classes in their program. Gerald guides them to creating simple frequency graphs using x’s to show the total of responses across an x-axis like this:

Smoke at least a pack of cigarettes a day:

	X
	X
X	X
X	X
X	X
X	X
X	X
X	X
X	X
X	X
X	X
X	X
X	X
<u>X</u>	<u>X</u>
Yes	No

From this simple frequency chart, students determine what categories yielded more than half of responses. They then dictate statements that Gerald captures on chart paper. Sample statements include such things as “More than 50% of those surveyed do not smoke at least a pack a day.” “Only 10 out of 22 smoke more than one pack a day.”

At the end of this series of lessons and activities, Gerald wants to review what students have learned about doubling. He wants to make sure that they understand the difference between halving and doubling and that they can use the terms as well. He references graphs that they have done and asks them to share what totals would have looked like if the numbers were doubled. This serves as an informal review and helps Gerald decide whether students are ready to move on to developing expertise with $1/2$ of a $1/2$ ($1/4$).

Hourly Wages

Ada is working with a small group of students who are struggling learning their “basic” addition facts. She knows that they have been struggling to learn their facts for years and does not want to make them feel inadequate by having them do the same drills as they did when they were younger. She feels that if she can show them how to apply their facts, the facts will “stick” with the students better, and they can practice their skills while learning new math concepts at the same time. She also hopes that by “playing” with addition, she can help her students make the leap to multiplication.

Because Ada knows that her students struggle with day-to-day issues related to math, she asks them about their comfort level with dealing with the finances related to their part-time jobs. None is comfortable and one shares that she just has to “trust what they give me.” Ada sees that there would be value in giving them real-life experiences adding numbers with decimals (for money), but she wants them to be more comfortable with addition and some simple multiplication facts first. That would make adding “messier” numbers easier for her students. She also thinks about designing lessons that begin to get her students to see patterns. This will help connect addition to multiplication and it will begin to lay the groundwork for algebra. Most importantly for the students, though, is that they will be building skills so that they begin to determine their own hourly wages and not have to “trust” others.

Ada brings in some play money so that her students can have concrete materials to work with. She talks about how students can figure out about how much they would make in a week based on their pay and the number of hours they worked.

WA Level 3: Use Math to Solve Problems and Communicate

WA Level 3 Indicators

M 3.1 Read, write, and interpret some common types of mathematical information such as

Numbers and number sense: whole numbers, monetary values and prices, benchmark fractions ($\frac{3}{4}$, $\frac{1}{10}$), decimals (.25, .50, .75, .10) and percents (25%, 75%, 10%, 100%).

Patterns/Functions/Relationships: simple patterns, probability and proportions (1:4, 4:1); simple decimal/fraction conversions and equivalents.

Space/Shape/Measurement: commonly used standard units of measurement, common geometric shapes, and the concept of "area".

Data/Statistics: simple ways to interpret and represent data (tables, bar graphs with and without gridlines, line graphs and pie graphs).

M 3.2 Recall and use mathematic procedures such as addition, subtraction, multiplication and division on whole numbers, benchmark decimals and fractions (with or without use of calculator), grouping, comparing 2 numbers, and basic estimating; and measure length, weight, and areas of standard and non-standard shapes using tools calibrated with whole numbers and benchmark fraction and decimal equivalents (rulers, manipulatives).

M 3.3 Evaluate the degree of precision needed for the solution.

M 3.4 Define, select, and organize simple data, and measure with appropriate tools, describe patterns, and/or use computational procedures effectively to solve a problem and to verify that the solution is reasonable.

M 3.5 Communicate the solution to the problem orally, in pictures, or in writing.

Show Fluency, Independence, and Ability to Perform in a Range of Settings

Adults performing at Level 3 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish simple, well-defined, and structured math tasks in a range of comfortable and familiar, or highly structured, settings.

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models

Teaching and Learning Objectives

To support performance of the Standard at this level, instruction should be offered in the context of real-life purposes, texts, and activities, and should, across the four strands, address the integration of knowledge, skills, and strategies such as those below:

Number Sense Strand

3A1 Read, write, and interpret common types of mathematical information:

- a. Read, write, and compare whole numbers up to 10,000
- b. Read, write, and compare whole numbers and benchmark fractions ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{10}$), decimals, and percents
- c. Recognize and use equivalent forms of benchmark fractions (such as $\frac{2}{4} = \frac{1}{2}$)
- d. Understand and use place value (to the nearest hundredth as in monetary values)
- e. Demonstrate and understanding of and recognize and apply simple negative numbers in real contexts (such as thermometers)

3A2 Recall and use mathematical procedures:

- a. Estimate with whole numbers
- b. Group, compare
- c. Add and subtract (with or without calculator)
- d. Demonstrate an understanding that multiplying by a simple fraction is the same as dividing by the whole number in the denominator (such as $\frac{1}{2} \times 10$ is the same as $10 \div 2$)
- e. Demonstrate an understanding that finding the root and square root are related

3A3 Determine degree of precision, rounding to the nearest 10, 100, 1000, 10,000

3A4 Use computational procedures effectively to solve real-life problems:

- a. Accurately add and subtract whole numbers
- b. Multiply and divide three digit whole numbers using effective strategies
- c. Determine parts of a whole using benchmark fractions such as $\frac{3}{4}$ and $\frac{1}{10}$
- d. Understand and use simple strategies for determining benchmark decimals (.25, .50, .75, .10)
- e. Understand and use simple strategies for determining benchmark percents (25%, 50%, 75%, 10%)

- f. Articulate the relationship between benchmark fractions, decimals, and percents (such as .50, 50%, $\frac{1}{2}$)
- g. Can apply proportions such as 4:1, 1:4
- h. Check for reasonableness of answer using estimation and benchmarks

3A5 Communicate results orally, in pictures, entries in a chart, and in writing

Patterns, Functions, Relationships Strand

3B1 Read, write, and interpret simple patterns, functions and relationships:

- Recognize, describe, and generalize for a rule for patterns
- Recognize and develop repeating patterns and generalize the relationship
- Generalize the relationship between quantities in a table of amounts
- Create simple expressions or formulas from real-life situations
- Create and apply very simple formulas from tables with one arithmetic step

3B2 Read and interpret symbolic information:

- Use exponents to show repeated multiplication from simple amounts (such as $3^2 = 9$)
Read and write operations using algebraic notation (parentheses or \cdot for multiplication and $/$ for division)
- Evaluate one-step equations

Space/Shape/Masurement Strand

3C1 Read, write, and interpret simple types of information related to measurement:

- Recognize and use commonly used standard units of measurement to the nearest half
- Recognize and describe common two- and three-dimensional shapes, including angle descriptions
- Identify right angles
- Describe informally the difference between square units (such as square inches) and linear units (such as inches)
- Use direction, distance, labels, simple scales, and symbols to read and use maps and plans
- Read and compare positive and negative temperatures on a Fahrenheit thermometer

3C2 Recall and use mathematical procedures:

- Solve problems involving money, including decimals
- Measure with a ruler to the nearest half and quarter
- Use appropriate tools to measure to the nearest half unit
- Demonstrate an informal understanding of the difference between inch and centimeter, yard and meter
- Make simple conversions within the same measurement system (such as inches to feet, cm to m)

Data and Statistics Strand

3D1 Read and interpret data and statistical information:

- Extract discrete information from a list, table, or bar graph
- Make numerical comparisons about relative values on a bar graph (such as “greater than” or “less than” or “about twice as much”)
- Identify the shape, range, median, and mean of data
- Make statements and simple comparisons based on data representations
- Demonstrate an understanding that bar widths and scale can distort interpretations of data

3D2 Collect, organize, and represent data:

- Collect and organize data based on posed question and organize in a reasonable manner
- Display categorical data in a simple bar graph or line plot
- Verify that data represented is the actual data collected

3D3 Recall and use basic probability concepts:

- State probability in a ratio fraction
- Find the probability of a single outcome in a simple concrete situation with a very limited number of possible outcomes

WA Level 3 Examples of Proficient Performance

Adults performing at Level 3 can select, record and work with simple mathematical information to accomplish a variety of goals, such as:

- Stay within a set budget while shopping for work clothes
- Determine the area of a room by grouping the square units within the shape and performing repeated addition
- Describe a simple equation for determining weekly pay based on a consistent, predictable pattern, such as \$5 per hour
- Review a restaurant check for a group of 5 people to see if there are errors; estimate the sales tax on the meal
- Measure the approximate number of feet of baseboard that will be needed for a room
- Design a garden with at least 2 rectangular flowerbeds; cost out fencing and plants using a catalogue
- Survey a group on the topic of your choice and create a bar graph to display data; explain findings
- Listen to 2 weather forecasters and keep track (by graphing) of the actual high and low temperatures each day for a week to see which forecaster is more accurate
- Use a calculator to add total receipts from 6 cash registers
- Quadruple a recipe for chocolate chip cookies for a community bake sale

Examples of a Person Performing at WA Level 3:

A Perfect Gift at a Perfect Price

Bryan is hunting for the “perfect” gift for his girlfriend. He wants to buy her a bracelet that is good quality but not too costly – quite a challenge! He has been watching the local jewelry stores for sales. He looks at a flyer that advertises “up to 75% off”. In order to figure out about how much he will have to pay for any of the bracelets on sale, he knows that he should first round to the nearest whole number. He also knows that he can adjust his whole number to make it easier to do calculations on.

He sees a bracelet that is regularly priced at \$99.95 that is 25% off. Mentally, he rounds \$99.95 to \$100, and then divides that by 4 because he knows that 25% is the same as $\frac{1}{4}$. He remembers that $\frac{1}{4}$ of a dollar is \$0.25, so $\frac{1}{4}$ of \$100 is \$25. If he saves \$25, then the bracelet must cost about \$75. He then sees another bracelet that is also 25% off. This bracelet normally sells for \$129.95. When he rounds to \$130 and tries to divide by 4 in his head, he has a remainder so he thinks about rounding \$130 to \$120 since 4 goes into 12 evenly. He figures 25% of \$120 is \$30. He remembers that the \$30 is the amount he will save, not the cost of the bracelet on sale. He counts by tens from 30 to 120 for a total of \$90, so he figures the bracelet should cost somewhere around \$90. He then uses a calculator to get a better sense of what the actual sale price for the bracelet is.

Making Tough Decisions

Bronson decides it is time to be on his own. He does not like living at home, but has not made enough money to be able to afford his own place. He recently got a raise so is now earning \$8.50 an hour. He works about 30 hours a week, so he multiplies $30 \times \$8.00$ to get an estimate on the low side of his total weekly earnings (He knows that \$8.00 is less than his hourly wage of \$8.50, but figures it is safer since there might be weeks when he doesn't work 30 hours.) He figures that $\$8 \times 30 = \240 . He knows that he has to pay taxes, which he thinks is about 25% of his earnings. He divides \$240 by 4 to figure out that he would have to pay about \$60 in taxes. That leaves him about \$180 a week. In a month, he would make between \$600 and \$800 a month. [He figured this out by reasoning that \$180 is more than \$150. $\$150 + \$150 = \$300 \times 2 = \600 . \$180 is less than \$200. $\$200 \times 4 = \800 .]

He thinks \$600 - \$800 is enough for him to find his own apartment. As he looks in the want ads, his eyes stray to the car ads. His old used car is not very reliable and he has to put a lot of money into it. He considers what \$800 a month in car payments would give him. He sees a car that interests him – especially because there is “no money down” and payments do not begin for three months. He looks at the price - \$14,999 – and thinks about how many years he would have to pay on the car based on \$800 a month. He calculates that $\$800 \times 12 = \9600 . Two years times \$9,600 is much more than \$14,999.

Then he reflects on his previous experiences with car buying and the conversations he has had with others about interest rates . . . and car insurance ... and . . . and. He realizes that he needs to talk to others to find out just what it would cost per month for insurance if he buys a newer vehicle and just what payments would look like for a car costing about \$15,000.

He finally decides to continue living at home for the time being. Maybe by living at home he could save money to buy a car. Hmm??

Examples of Teaching and Learning at WA Level 3:

(Exit point for National Reporting System Low Intermediate Basic Education)

Shaping Up!

Adrienne is working with a group of students who are struggling learning their “basic” multiplication facts. She asks her students when they feel they could have solved a math problem more readily if they had known more math. Students give various examples, and when one student mentions having a hard time figuring out the cost of materials needed for putting down his deck, Adrienne sees this as an opportunity to talk about the concepts of area (and later, perimeter) while working on addition and multiplication facts. She also knows that, before she can address the idea of decking, she needs to be sure they understand the concept of area.

She first finds out what students already know about area. She builds on the students’ comments about the flooring for the deck by asking if they know of other examples of things that take up flat space. They struggle at first, but then seem to be full of examples of things that represent area: ceiling tiles, carpeting, mirrors, and pictures.

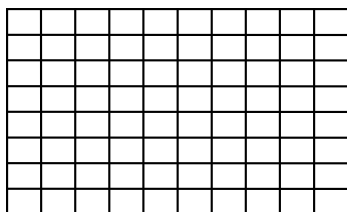
Because Adrienne knows it was important to start with concrete examples, she asks students to work in small groups to consider what size they might want to build a deck. She suggests that they might want to begin with a simple design since it would be their first deck that they would “build” together. Students seem unsure of just how large various deck sizes might be, so she asks for a size that someone had heard about.

One student offers, “An 8 x 10 deck.” She asks if they know what the 8 and the 10 represents. Students in general seem to know that 8 and 10 represented the number of feet in the length and width of the deck. She then asks for a pair of students to volunteer to measure out an 8 x 10 deck on the classroom floor using masking tape. They use a foot ruler to measure each of the strips of tape and other students join in to ensure that the corners are fairly accurate. She probes to make sure they understand that the area is represented by the space within the tape, not the length of each strip of tape (which would represent the perimeter).

She asks them if they know what the area of the deck is. They don’t seem to know, so Adrienne suggests that they draw a similar shape on their graph paper using each length of the grid as one foot. The graph paper is useful because students are able to readily see the actual square units created by the 8 x 10 rectangle. Students intuitively sense to count the total number of squares that are enclosed in the shape. They see 80 square units. When asked how they figured that out, one student says, “I counted all the squares.” Another mentions, “I started counting, then realized that there were 10 in each row across, so I just counted by tens.” Yet another student reported, “I counted them first, then realized that I could just multiply the two sides together. 8 x 10 is 80.”



She then asks whether the same would be true with the 8 foot by 10 foot rectangle taped on the floor. Students decide to check it out. They use the masking tape to add seven 10-foot lengths running parallel in order to show eight rows running down the deck.



They then tape nine 8-foot lengths parallel to the 8-foot sides to show that there are ten feet running across the deck.

They immediately see that there are 80 square units in the shape on the floor. Adrienne asks them what the size of each of the squares is. After students ponder a bit, several announce that each square is a 1 foot by 1 foot square; therefore, there are 80 square feet in an 8 foot x 10 foot area.

Before going back to the original question about the cost of the basic materials for decking, Adrienne wants to check that students understand the concept of area and some strategies for figuring it out. She describes some different shapes (all rectangles, all “friendly” whole numbers) and asks them to draw the shape on graph paper and then determine the area of each shape. She asks the students to keep their drawings because she knows she will revisit these to discuss the difference between the area and the perimeter of rectangles.

She then comes back to the original question about how to determine the cost of materials. She knows that decking boards comes in widths that are just under 6” wide, so two boards would be equivalent to about one foot. She knows that different contractors determine costs differently: some by the total area and some by the total board length (to determine just the cost of the decking boards themselves – usually the most costly item).

She decides that she wants this to be as true to life as possible so she has them do a bit of research to find out the cost of decking boards at various lumberyards and home improvement centers. She also encourages them to closely examine decks to discover what they can about the layout of them.

When they return with their prices and observations, she tells them they are ready to find out the cost of at least the decking boards. They talk about what they observed about decks, including the fact that boards are not butted up against each other without any gaps. That leads to a discussion about how it takes about two decking boards to make a width of one foot. They then revisit their earlier drawing (and taped replica) to predict the number of board feet that would need in order to build an 8’ x 10’ deck.

Students use masking tape to actually show what an 8 x 10 deck would look like with 5 3/4 inch decking boards. Then they use their graph paper to visualize the deck with each of its boards. They use different strategies to determine the total board length needed and compare their answers and their strategies. Then someone asks what would happen if they turned the boards in the opposite direction. Would the amount of board lengths be the same? Would it cost more? Less? The same?

Adrienne knows she could readily answer this question, but she also knows that students need to learn how to answer those questions themselves. She suggests that they create the shape on the floor (and on their graph paper). She smiles as she understands that the more students learn, the more they ask their own questions and want to learn how to figure out the answers to those questions.

Graphic Design

Darlery and her students have been working on benchmark percents, including 50%, 25%, 75%, and 10%. Students have been bringing in examples of flyers from stores that have sales involving percents and they have enjoyed being able to figure out for themselves the estimated sales prices of various items.

While Darlery feels comfortable that they can use benchmark percents in shopping situations, she knows that students do not automatically transfer their understanding to different situations. She asks them where else they see percents. One student mentions that he has seen percent in a newspaper article that also showed a circle chart. Other students then reflect on their struggle with reading the various charts that appear in print materials. One student even comments, "I just skip the graphs."

Darlery encourages her students to talk about their experiences with graphs so she can discover what students already know about graphs as well as what misperceptions they might have. One student mentions that, "with graphs that use lines, it's always good when the line goes up." Darlery tucks this response in the back of her head so she can be sure to address it with an example or two later. For now, it seems, from their conversations, they need some concrete experiences working with simple graphs so that they will develop a strong foundation for building on graphs representing several sets of data.

Ultimately, Darlery wants to have students create their own circle graphs, but she thinks it would be easier to first begin with bar graphs, made from frequency charts. Darlery wants to keep the total data to be plotted small so that students will not be overwhelmed just by the numbers themselves. She suggests that they design a survey and ask other adult learners in the program some questions that are of interest to them.

Students discuss issues that relate to the program as a whole. For example, they discuss the hours and days of the week that the program offered services. Someone asks, "Why don't we offer classes on Saturday so people who work can attend?" Students think that is a good question to ask and they soon have a list of questions to pose. Some of the questions are designed to yield yes/no responses; others give interviewees a range of choices; and yet another question is open-ended.

Once students collect data, she makes multiple copies of their results, then breaks the students into triads to decide how to organize their results. Students have no difficulty organizing their data for the yes/no responses. They also have no trouble grouping the range of choices; however, the open-ended question stymies the students. Darlery suggests that they organize the data in ways that are meaningful to them. She does not want to dictate how the data should be represented. She also is hoping to get a variety of organizational strategies since this would yield different graphs for the same data – an important point that she hopes to impress upon her students. For example, one question students ask is "What time of day would you prefer class to begin?" The responses include the following:

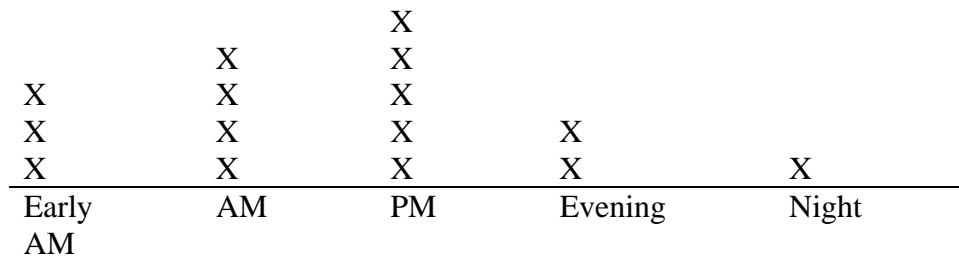
6AM - 1, 8 AM – 2; 9 AM – 2; 10 AM – 2; 1 PM – 3; 4 PM – 2; 6 PM – 2; 11 PM – 1

She wants to see whether students will organize the times by morning, afternoon, evening, or by another organizational strategy that might include AM vs. PM times, or would such "outliers" be considered a lumped category of "other".

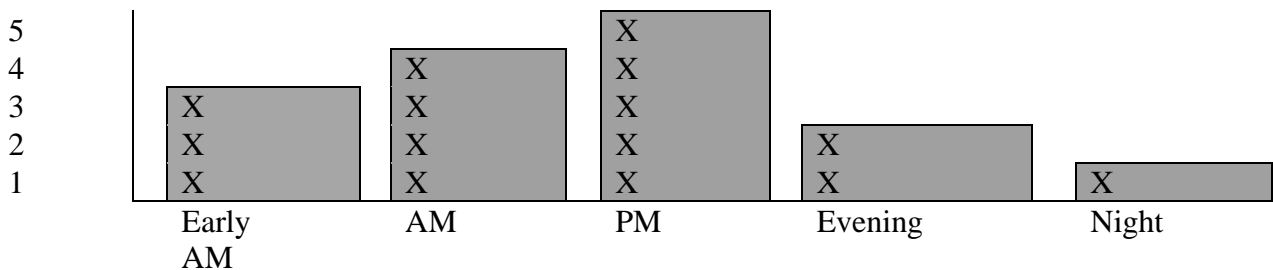
After students have organized their data, Darlery asks what they think should happen next. Students think the data should be put into charts or graphs. Again, Darlery takes time to ask them what should be included in a graph. She also asks them what they know about the different types of graphs. Because of

the range of experiences of her students, she suggests that they bring in as many examples as they can of different graphs. They also look in math, social studies, and science workbooks for examples. From these examples, students are able to brainstorm a list of requirements for a good graph. They are not as clear about whether different graphs serve a different purpose but students did see that data showing change over time is represented in line graphs.

Darlery models how to create a bar graph of the data from the question about preferred time for classes to begin. She uses the following categories after much discussion and debating with students: Early morning: 3; Morning: 4; Afternoon: 5; Evening: 2; Night: 1. With the students' input she creates the following frequency graph with x's representing each response and puts the graph on a large piece of chart paper:



She then creates a bar graph by closing in the x's and adding an x- and y-axis (without calling them as such . . . yet).



Next, she cuts out each of the bars (after being sure that it is colored and labeled), being sure to keep the length of the bar accurate. She tapes each of the bars together into one continuous strip and then makes a circle with the long strip. As students watch and assist, she lays the strip down and marks the division of each strip, thereby creating a fairly accurate representation of the data in a circle graph.

Then each group chooses one other set of data to represent and follows the same procedure: creating a frequency graph using just x's, then boxing in the x's to create a bar graph with labels, then cutting out each bar (with labels) and taping all the pieces together into a circle.

After coloring and coding their circle graph, Darlery encourages her students to talk about what the circle graph communicates vs. the bar graph (circle graph shows how each category relates to the total number of items in all the categories while the bar graph allows them to see how each individual category compares to another category). They also compare the individual bars to the total and look for any that might be at least 50% of the data. 25%. 75%. Students then look to see how these percentages are reflected in the circle graph by looking at whether the slice is about half. About a quarter of the total.

Finally, Darlery makes connections back to the students' previous work in which they determined what 50% of a total cost was. She asks them to explain how 50% is used in each example. She also asks them to explain in their own words some strategies for determining 50% (and 25%) of an amount, and whether they used the same strategy in each situation, or whether they changed strategies when the situation changed.

WA Level 4: Use Math to Solve Problems and Communicate

WA Level 4 Indicators

M 4.1 Read, write, and interpret a variety of common mathematical information such as

Numbers and number sense: monetary values, extensions of benchmark fractions ($\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{5}$, etc), decimals, and percents (15%, 30%, etc.).

Patterns/Functions/Relationships: patterns and simple formulas (such as $d=rt$, $A=lw$);
Space/Shape/Masurement: standard units of measurement including fractional units and benchmark angle measurements (90 degrees, 360 degrees, etc), geometric shapes including shapes containing a combination of common shapes, concept of pi, and concept of converting between units of measurement.

Data/Statistics: ways to interpret and represent data (tables and graphs with scaling, basic statistical concepts such as range, mode, mean, and median).

M 4.2 Recall and use a good store of mathematical procedures such as estimation, rounding, multiplication and division (with and without use of a calculator), adding and subtracting, multiplying and dividing common fractional amounts and decimals, measure length, weight, area and circumference using tools calibrated to varying degrees of precision and converting units of measurement as appropriate.

M 4.3 Evaluate the degree of precision needed for the solution.

M 4.4 Define, select and organize a variety of common mathematical data and measure with appropriate tools, describe patterns, and/or use appropriate procedures effectively to solve a problem and verify that the solution is reasonable.

M 4.5 Communicate the solution to the problem orally, with visual representations, in writing, by entries in a table or appropriate graph, or with basic statistics (range, mode, mean, median).

Show Fluency, Independence, and Ability to Perform in a Range of Settings

Adults performing at Level 4 can easily *select and apply the knowledge, skills, and strategies at this level to independently accomplish well-defined and structured math tasks in a range of comfortable and familiar settings.*

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models.

Teaching and Learning Objectives

To support performance of the Standard at this level, instruction should be offered in the context of real-life purposes, texts, and activities, and should, across the four strands, address the integration of knowledge, skills, and strategies such as those below:

Number Sense Strand

4A1 Read, write, and interpret a variety of common mathematical information:

- a. Read, write, and compare whole numbers (to billions)
- b. Read, write, and compare extensions of benchmark fractions, decimals, and percents (such as $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{100}$)
- c. Convert extensions of benchmark fractions, decimals, and percents to their equivalents
- d. Recognize and apply negative numbers in real contexts (such as thermometers, winning and losing money)
- e. Read, write, and compare decimals to three places

4A2 Recall and use a good store of mathematical procedures:

- a. Estimate with benchmark fractions, decimals, and percents
- b. Multiply and divide accurately (with and without calculator)
- c. Add and subtract common fractional amounts
- d. Calculate percent increase or decrease with friendly numbers

4A3 Determine degree of precision (such as to the nearest billion or to the nearest tenth or hundredth)

4A4 Use computational procedures effectively to solve real-life problems:

- Use extensions of benchmark fractions ($\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{5}$, etc.), decimals, and percents (15%, 30%, etc.)
- Add, subtract, multiply, divide with any whole numbers using effective strategies
- Add and subtract decimals up to three places
- Use and demonstrate understanding of order of operations in multi-step problems
- Use benchmark fractions, decimals, and percents to check for reasonableness of answer

4A5 Communicate results orally, in pictures, entries in a table or graph, with basic statistics (range, mean, median, mode), and in writing

Patterns, Functions, Relationships Strand

4B1 Read, write, and interpret a variety of common patterns, functions and relationships:

- Recognize and describe patterns and apply simple formulas
- Recognize and develop repeating patterns and generalize the relationship with a table, rule, or simple formula
- Create simple expressions or formulas from real-life situations
- Create and apply simple formulas from tables and create simple linear graphs

4B2 Read and interpret symbolic information:

- Use exponents to show repeated multiplication in real contexts
- Evaluate simple formulas and expressions
- Apply order of operations to solve equations and expressions
- Write statements of equality and inequality (such as $3 > 4 - 3$)

Space/Shape/Masurement Strand

4C1 Read, write, and interpret a variety of common mathematical information related to measurement:

- Measure and compare radius, diameter, circumference of a circle
- Use a protractor to measure angles
- Use scale to find distance between locations on a map
- Locate points on a globe or map using latitude and longitude

4C2 Recall and use a good store of mathematical procedures:

- Convert units of measure from one system to another using informal methods
- Use appropriate tools to measure to the nearest hundredth
- Find the volume of cubes

Data and Statistics Strand

4D1 Read and interpret data and statistical information:

- Make absolute comparisons about relative values on graphs and tables
- Identify the shape, range, mean, median, and mode of data
- Describe the effect of range on median and mean and which is most appropriate for the data
- Make simple comparisons to support or refute statements based on representations of data
- Demonstrate an understanding that scale and sample can distort interpretations of data

WA Level 4 Examples of Proficient Performance

Adults performing at Level 4 can select, record, and work with mathematical information to accomplish a variety of goals, such as:

- Figure a tip on a restaurant bill
- Balance a checking account
- Write a children's book on multiplication facts
- Determine how much flooring is needed for a room by multiplying the length times the width of the room
- Design a survey regarding a community issue, and collect and organize the results
- Develop a monthly budget
- Design a garden to get the most space with the least amount of fencing needed
- Decide which product to buy based on a comparison of nutritional information
- Figure estimated taxes

4D2 Collect, organize, and represent data:

- a. Collect and organize data based on posed question and represent the information in a table, list, line plot, or frequency table
- b. Display categorical data in a bar graph or circle graphs
- c. Convert bar graphs into circle graphs

4D3 Recall and use basic probability concepts:

- a. State probability in ratio fraction and compare to benchmark fractions
- b. Find the probability of a single outcome in a concrete situation

Examples of a Person Performing at WA Level 4:

Proportional Reasoning Strategies

Cody uses a variety of strategies to solve problems. For example, he generally uses mental math strategies with benchmark fractions, decimals, and percents (such as 50%, 25%, $\frac{1}{2}$, .5, 10%). He does a lot of rounding to estimate in order to get a ballpark answer. For example, when he hears that 47% of the people voted against an issue, he thinks to himself, “47% is close to 50%. 50% is the same as half, so just under half the people voted against the issue.”

For much of his everyday problem solving that involves proportional thinking, Cody uses informal strategies. When he sees that two cords of wood stacked take up four pallets, he knows that four cords would take up eight pallets by looking at the ratio of cords to pallets. While he does not necessarily consider what he is doing as using ratios and proportions, he thinks, “For every two cords, I need four pallets. Four cords of wood is double two cords, so I need twice as many pallets as I would need for two cords. I need eight pallets.”

He is planning to travel to England soon and is trying to figure out how to make comparisons with the U.S. dollar and the British pound. The last time he checked the exchange rate he found this: 1 US dollar = .544425 GBP. At first, the messy number made him a bit anxious because his informal strategy didn’t seem to work for these messier numbers. Then he remembered another tool for proportional reasoning – the cross-product rule. Using this tool would help him keep dollars vs. pounds straight.

He thinks he would like to exchange \$250 into pounds. He first does some reasoning by considering that the dollar is worth about $\frac{1}{2}$ pound. So his \$250 will not get him many pounds – about half of 250, or 125. Because he wants to be more accurate than this, he sets up a proportion using the original conversion as a ratio:

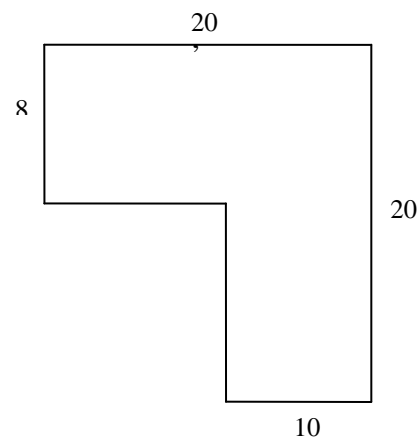
$$\$1/.544425\text{£} = \$250/P$$

He knows he needs to cross multiply, which he does, using a calculator: $P = .544425 \times 250 = 136.0625\text{£}$. He now knows how many pounds he can get for his \$250.

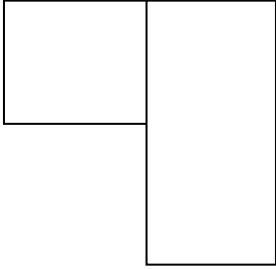
He also realizes that, if the exchange rate remains similar and a dollar equals about half of a pound,, he will have to think “double” for anything that he sees priced in pounds. He knows that if he wants to buy a hamburger costing 3.50£, he should not think that is a good price compared to a hamburger in the U.S. Instead, he should mentally multiply 3.50£ times 2 to get a better estimate of what it costs in U.S. currency.

Floored!

Morgan wants to redo her living room floor. She knows that she has to measure the size of the room and plug in the dimensions into the formula for area. However, her living room is not a rectangle; it is L-shaped. She sketches a picture of the shape and labels each of the dimensions as shown in the picture on the right.



She realizes that she actually had several strategies she can use to figure out the area of the room.

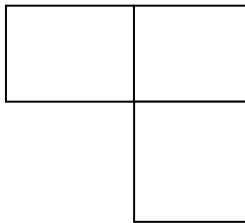


She can create a smaller rectangle (8' x 10') and a longer rectangle (10' x 20') such as she did in her drawing on the left.

She could also create a wider rectangle (8' x 20') and a smaller rectangle (12' x 10') such as in the drawing on the right.



She could even create three small rectangles: 8' x 10', 8' x 10', and 12' x 10', as shown on the left.



She does some quick calculations for each set of measurements. For the first drawing, she figures: $80 + 200 = 280$ sq. ft. For the second drawing, she calculates: $160 + 120 = 280$ sq. ft. And, for the three small rectangles, she finds that the total area was still the same: $80 + 80 + 120 = 280$ sq. ft. By doing these quick

calculations various ways, she feels comfortable about her calculations.

She has been looking at some special tiles that are 1' x 2' rather than the usual 1 ft. sq. tiles. She uses a combination of drawing and calculating to determine that she would need 280 tiles if she used 1' x 1' tiles and half as many (140) if she were using tiles that were 1' x 2'.

Examples of Teaching and Learning at WA Level 4:

(Exit point for National Reporting System High Intermediate Basic Education)

Which Is the Better Job?

Phyllis has a new group of students who need to “brush up” on the math skills. In conversations she has had with them, she discovers that, for the most part, they do not like math and barely survived the algebra courses they took in high school. Phyllis knows that algebra is a gatekeeper for many of her students and wants to ensure that they have a firm grounding in algebraic reasoning. She does not want to just teach them rote rules and formulas but rather wants them to be able to see how they might already be thinking algebraically and how this thinking can affect how they solve problems in their daily lives.

Her students have experience in solving simple equations and using basic formulas, but they have no experience creating their own formulas or equations, nor do they see any value in learning algebra. Phyllis knows she has a challenge on her hands, so she listens for opportunities to connect to her students’ own situations.

She hears a couple of her students talking about a potential job offer. She sees this as an opportunity to have them apply their algebraic reasoning to a real situation. She asks her students to consider which is the better part-time job offer: one in which the employee makes \$12 an hour or one in which the employee makes \$10 an hour but gets a sign-on bonus of \$200. She asks students to first discuss the question in triads and then share their responses.

She then asks them to think about the pattern for making \$12 an hour. She sees blank looks on her students’ faces, so she decides she needs to model for them. She realizes her students, to a great extent, have had math experiences limited to workbooks which deal with lots of “naked” numbers that have no context and only ask students to perform operations.

She wants to use simple numbers so students see the big ideas rather than struggle with the numbers themselves, so she uses the rate of \$6 an hour. She asks them how much she would make in two hours at that rate. Students readily give her the correct response. She pursues this line of questioning pushing students to

figure out the total wages after 10 hours; 20; 100. She asks them each time how they figure out the answer and they respond with, “We multiplied.” She then asks them for a rule to use to determine her total wages no matter how many hours she works. They say that she would need to multiply the number of hours worked times the hourly wage - \$6.

She then focuses on helping them understand how to write out their thinking using algebraic symbols. She asks how to write the multiplication problem of ten times six and discovered that they tend to use only one way. She shows them how to write multiplication several ways: 10×6 ; $10(6)$; $(10)(6)$; and even $10 \cdot 6$. After a bit of practice writing different examples, she asks how they would write \$6 times an unknown number of hours. After some discussion, she helps students to see that N could represent any number of hours and that multiplication with an unknown amount could be written yet another way: $6N$. She then creates a simple formula as she talks aloud about the symbols: “ T for total pay = 6 times however many hours I work which is unknown right now, so $T = 6N$.”

Students seem relieved to discover how simple it was to translate their own words and thinking into algebraic symbols. They are ready to try another hourly rate using \$12. They work in groups to see that for every hour worked, they would earn \$12. To find the total amount earned, they would multiply the number of hours worked times \$12. Phyllis then asks for a rule for figuring out the total pay no matter

how many hours are worked. Students together write out their rules, which they then develop into a simple formula: T (total pay) = $12H$ (hours worked). They graph this on a sheet of chart paper using their pattern: 1 hour/\$12, 2 hours/\$24, etc. They see this is a straight-line graph.

She then asks them to consider how to create a formula and graph for the other job: \$10 an hour plus an incentive of \$200. For the most part, groups of students first plug in numbers to notice a pattern; they then arrive at a similar formula for the \$12 an hour job, noting, though, that there was always going to be an extra \$200. They develop this formula: $T = 10H + 200$. Again, they graph what the total wages would be based on their pattern of hours worked: 1 hour/ $10 + 200$ (210), 2 hours/220, etc.

Phyllis then asks them to compare the two graphs and determine what the difference in the two jobs really is. Students comment that the second job began at 200 rather than 0 on the graph. They talk about why this is so: the individual taking the job would get \$200 just by agreeing to take the job so the total number of hours worked would be 0. They notice that there is a point when the two lines cross. [Later Phyllis will use this as an opportunity to show the equation $12H = 10H + 200$ as the time when the two total earnings are the same. She will use this as a jumping off point to give students further experience in working with basic algebraic equations.]



She asks students to ponder what the intersection point on their graphs represents. They compare their total earnings for each job at the point of intersection at 100 hours and realize that it is the point at which both jobs would have an equal amount of total earnings. One student then comments on the two jobs based on their graphic representations, “If I don’t plan to stay long at the job, then the \$10 an hour job is best because I get the bonus.” Another adds, “And, if I plan to keep the job for a while – at least for 100 hours of work – then it makes more sense to take the \$12 job. I think someone is trying to trick us into taking the job with less money per hour because we’ll want the \$200.”

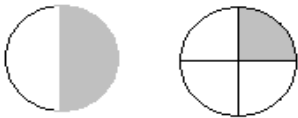
A Timely Lesson

Ada heard one of her students talking about wanting to get his GED so he could try to get another job. When she encouraged him to talk about his previous job, she discovered that he had been fired from his last job for supposedly “lying” on his time sheet. He explained that he did not intentionally lie, but he must have put down the incorrect times when he had to change minutes to fractions. Several students chimed in that this was an issue. Ada knew that she could develop some lessons around fractions and she could readily make the link to the GED Test.

Ada began by asking students to work in pairs to list all the fractions of hours that they already knew. She discovered that they were comfortable with 30 minutes as $\frac{1}{2}$ hour and 15 minutes as $\frac{1}{4}$ hour. That seemed to be the extent of their comfort level.

While still in groups, Ada asked the students next to determine how they knew that 30 minutes was the same as $\frac{1}{2}$ hour and 15 minutes the same as $\frac{1}{4}$ hour. While she knew that, ultimately, she needed to be sure they could quickly change minutes to fractions, she wanted to be sure they understood the concept, not just memorized a strategy or technique, so she encouraged them to draw pictures or create patterns to visualize the relationships.

Students then shared their results. A couple of groups had drawn a clock as a circle and colored in $\frac{1}{2}$ and $\frac{1}{4}$ of the circle to show that 30 out of 60 was half and 15 out of 60 was $\frac{1}{4}$. Another group had had previous exposure to fractions so were able to share how they reduced their fractions ($\frac{30}{60}$ and $\frac{15}{60}$) to simpler fractions.



$$30 \text{ min} = \frac{1}{2} \text{ hr} \quad 15 \text{ min} = \frac{1}{4} \text{ hr}$$

From the visual, Ada worked to help students understand that the fractions that they would be dealing with represented part/whole, with the whole being the entire number of minutes in an hour. She moved from the more common fractions – $\frac{1}{2}$ and $\frac{1}{4}$ – to less common fractions such as $\frac{1}{3}$ and $\frac{1}{6}$. These were still easy to work with and students could use manipulatives or drawings to illustrate what these represented in terms of minutes.

Students drew pie charts so that they could see what $\frac{1}{3}$ of an hour represented. The pie chart, for them, symbolized a clock, which they were very familiar with. Ada also encouraged them to look for patterns so she listed on the board the following ratios:

$$\frac{1}{2} = \frac{30}{60}$$

$$\frac{1}{4} = \frac{15}{60}$$

Because they have already had some practice “playing” with proportions, they knew to look across the pairs of ratios as well as within each ratio. They could see the 1 was half of 2; therefore, 30 should be half of 60. They noticed the relationship of 1 to 4 and the relationship between 15 and 60 (and they added $15 + 15 + 15 + 15$ to confirm that there were four 15’s in 60). Students could also see that 4 was twice as much as 2, but the number of minutes was half. They discussed this interesting pattern and took time to try out other sets of ratios to see whether the pattern always worked. They discovered that it did!

Once students were grounded in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{6}$, she asked students to give the rule for changing minutes to fractions. She then suggested that students keep their daily attendance for a week and then submit, at the end of the week, the total number of hours that they participated, including any fraction of hours. She knew this was a way for students to practice applying what they had just learned by keeping time similar to what might be expected on a timecard.

In the meantime, she would find out whether students could change minutes to decimal equivalents.

WA Level 5: Use Math to Solve Problems and Communicate

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables and algebraic models.

WA Level 5 Indicators

M 5.1 Read, write, and interpret a wide variety of mathematical information such as

Numbers and number sense:

money/expenses/prices, percentages, decimals and fractions.

Patterns/Functions/Relationships: patterns and formulas (such as $A=\pi r^2$).

Space/Shape/Masurement: units of measurement including fractional units, geometrical shapes including shapes containing a combination of common shapes, and concept of volume.

Data/Statistics: ways to interpret, represent and draw implications from data (graphs, tables, and simple forms of statistical analysis).

M 5.2 Recall and use multi-step mathematical procedures (such as keeping accounts) that involve whole numbers as well as fractions, decimals, and/or percents, and measure volume using tools with different calibrations.

M 5.3 Evaluate the degree of precision needed for the solution.

M 5.4 Define, select, organize, and integrate mathematical information of different types in carrying out procedures, describing patterns, and/or measuring with appropriate tools to solve the problem and to verify that the solution is reasonable.

M 5.5 Create appropriate visual or graphic representations such as charts, tables, graphs, etc. and clearly communicate the solution process and results orally or in writing to a variety of audiences.

Show Fluency, Independence, and Ability to Perform in a Range of Settings

Adults performing at Level 5 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish structured math tasks in a variety of comfortable and familiar settings.

Teaching and Learning Objectives

To support performance of the Standard at this level, instruction should be offered in the context of real-life purposes, texts, and activities, and should, across the four strands, address the integration of knowledge, skills, and strategies such as those below:

Number Sense Strand

5A1 Read, write, and interpret a wide variety of mathematical information:

- a. Recognize and use positive and negative numbers of any size
- b. Read, write, and compare fractions and mixed numbers and decimals

5A2 Recall and use multi-step mathematical procedures:

- a. Demonstrate an understanding of what happens when fractions are added, subtracted, multiplied, or divided
- b. Demonstrate an understanding of what happens when decimals are added, subtracted, multiplied, or divided
- c. Demonstrate an understanding of what happens when integers are added, subtracted, multiplied, or divided
- d. Calculate percent increase or decrease

5A3 Determine degree of precision

5A4 Use computational procedures effectively to solve real-life problems:

- a. Add, subtract, multiply, divide with decimals
- b. Add, subtract, multiply, divide using fractions
- c. Compute with percentages

5A5 Communicate results, including the solution process, orally, in writing, in tables, graphs, etc.

Patterns, Functions, Relationships Strand

5B1 Read, write, and interpret a wide variety of patterns, functions and relationships:

- a. Recognize, describe, and extend patterns and apply formulas
- b. Demonstrate an understanding that patterns and relationships can be represented with a formula, a table, or a graph
- c. Develop algebraic expressions, generalizations, or formula for simple number patterns
- d. Describe general shape and qualities of linear and simple non-linear graphs

5B2 Read and interpret symbolic information:

- a. Solve problems in two-step equations
- b. Evaluate formulas
- c. Solve linear equations

Space/Shape/Measurement Strand

5C1 Read, write, and interpret a wide variety of mathematical information related to measurement:

- a. Use the language of metric unit prefixes to describe real-life measurements
- b. Demonstrate an understanding of the coordinate graph system, including ordered pairs

5C2 Recall and use multi-step mathematical procedures:

- a. Use ratio and proportion in familiar settings involving scale drawings or maps
- b. Develop and apply formulas for perimeter, area, and volume
- c. Apply the knowledge of 180 degrees in a triangle to solve problems
- d. Use the Pythagorean theorem to problems involving right triangles
- e. Determine the slope of a line, the y-intercept, and the intersection of two lines
- f. Predict the impact of changes in linear dimensions on the perimeter, area, and volume of a figure

Data and Statistics Strand

5D1 Read and interpret data and statistical information:

- a. Make absolute comparisons about relative values on graphs and tables
- b. Infer meaning from gaps, clusters, trends, and comparisons of data
- c. Predict consequences related to data interpretations
- d. Identify the shape, range, median, mean, and mode of data
- e. Describe the effect of range on the mean and median and which is most appropriate for the data
- f. Make statements using interpretations from data
- g. Demonstrate an understanding that sample size and representation, scale, and categorization of data can distort interpretations

5D2 Collect, organize, and represent data:

- a. Collect and organize data based on posed questions
- b. Choose appropriate representations to display data collected
- c. Collect comparative data on a single question (such as responses grouped by gender vs. by ethnicity)
- d. Display comparative data on a double bar or line graph

5D3 Recall and use basic probability concepts:

- a. State probability in different formats: ratio fraction and percent
- b. Find the probability of independent and dependent events

WA Level 5 Examples of Proficient Performance

Adults performing at Level 5 can select, analyze (by categorizing and comparing), and work with mathematical information to accomplish a variety of goals, such as:

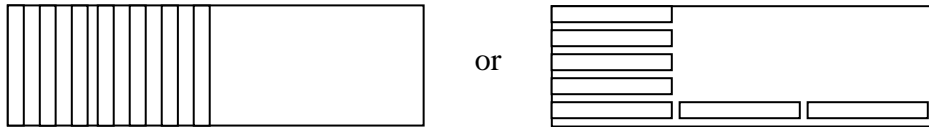
- Design and measure shelves for a closet
- Keep track of monthly income and expenses
- Design a garden and determine the amount of fertilizer, fencing, and plants needed
- Develop a yearly budget and illustrate it by creating a graph
- Create a presentation using assorted charts and graphs to influence a committee
- Choose which car to buy based on information such as down payment required, monthly installments, and insurance costs
- Record product measurements on a chart to monitor whether the process is in control
- Estimate the gallons of water in an irregularly shaped garden pond in order to purchase the right amount of pond clarifier

Examples of a Person Performing at WA Level 5:

DVD Shelf

Joyce wants to build a shelf to hold her DVDs but has very limited space. She finds a bit of wall space that is unused and figures that a shelf would be perfect there. She wants the DVDs to fit snugly without a lot of waste. She realizes that she has to consider several dimensions: the height of each shelf in order to fit the DVDs in snugly (yet make it easy enough to pull them out) as well as the overall height of the entire shelf; the depth to make the shelf so neither DVDs nor the shelf itself sticks out too much; and the width of each DVD and the shelf itself so it will fit into the wall space.

Joyce first measures the shape of one of her DVD cases: 5 1/2 inches long, 4 7/8 inches tall, and 3/8 inch wide. In measuring all the dimensions of the DVD case, she finds that she could possibly plan to stack her DVDs on end or one on top of another, which means that she would have to design the height and depth of the shelf accordingly.



Joyce also knows that, while her measurements

have to be exact in cutting each board length, she can round the dimensions of the DVD case to make it easier to plan (and because she does want extra room between the top of a DVD and the next shelf).

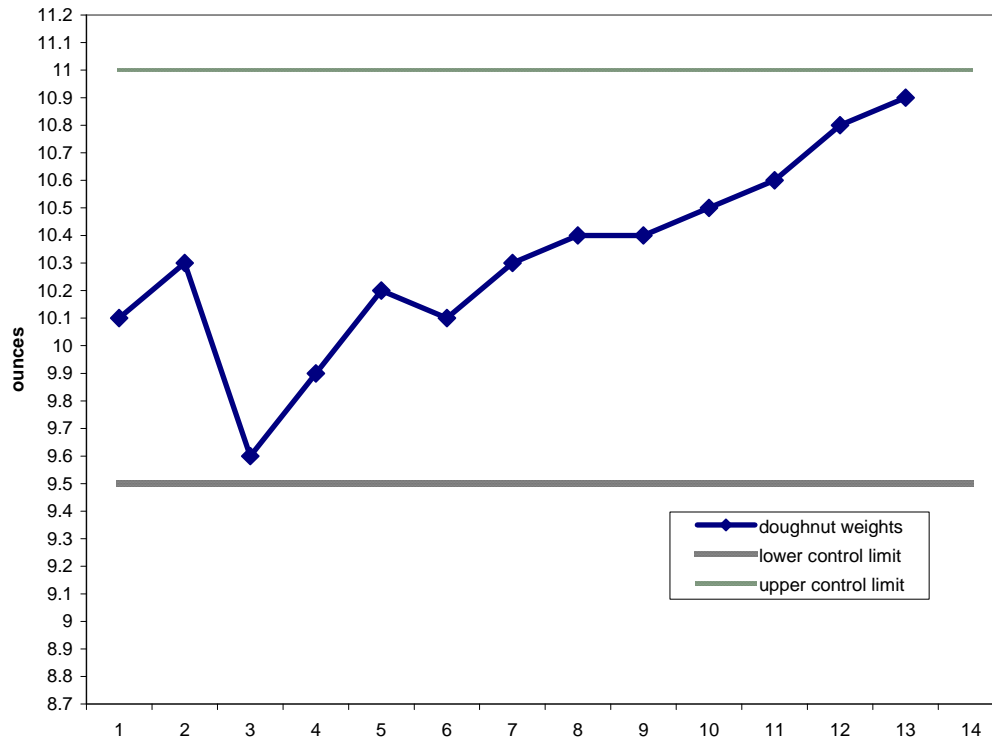
She uses a combination of rounding, drawing pictures, and using paper and pencil calculations to determine the overall size of the shelf and then the spacing between shelves.

Weighing In

Theresa is monitoring a doughnut making process at work. Every 15 minutes she takes a sample of doughnuts and weighs them. She then records the weight on a line graph. Her supervisor has told her to let her know if she sees “anything”. Theresa knows this means that she needs to look for patterns or trends in the data she is collecting and representing on a graph. She also needs to look for spikes or other irregularities.

She has weighed 13 samples and recorded each set of weights on a chart template developed by the quality control manager. Her graph looks like this after her 13 samples:

Doughnut Weights



She remembers that her supervisor asked her to not just record the measurements but to also look for possible patterns. She notices that there seems to be an upward trend in the weights of the doughnuts. While the weights are not quite at the upper weight limit of 11.0, she notices that the weights are getting very close. She realizes that, if the trend continues, the doughnut weights will be over the allowable limit. She makes a decision to contact her supervisor so he can determine whether adjustments need to be made in order to keep the process in control.

Examples of Teaching and Learning at WA Level 5:

(Exit point for National Reporting System Low Adult Secondary Education)

Up in Smoke

Students in Ken's class recently read an article about smoking that included a number of charts and graphs communicating information about cigarette use in one region of the country. They discussed the accuracy of the data presented. Ken suggested that they do their own research to see how their region compares to that listed in the article.

Ken asked the students to think about exactly what they wanted to know. They discussed the process for developing a survey that would be effective in gathering the kinds of information that they needed. The students wanted to ask open-ended questions. . To prepare them for organizing information that comes from open-ended questions, Ken suggested that they practice by posing a couple of questions on different topics and then collecting data just in their own classroom.

They asked each other questions, such as "Which cigarette brand has the coolest name?" and "How old were you when you first 'sneaked' a puff?". Once they had all the data and started trying to organize it, they realized that it could be very challenging to figure out how to organize data when the responses are completely open-ended.

Students then decided that their survey on smoking ought to include one or two open-ended questions along with several that either were yes/no or gave a range of options.

The next challenge that Ken knew they needed to consider was the sampling method. He and the students talked about who should be surveyed and why. Through the discussion, they decided that, for this activity, they would focus on just their own adult education program and compare their survey results to the data from the original article. They did talk at length, however, about sampling and wondered whether the sample collected for the article was representative or not. Ken saw that students were becoming more critical about the various charts and graphs that they were exposed to. They made comments such as, "I always wondered where they got those numbers from, but now I need to think about whether they asked a variety of people or not." Another student exclaimed, "So, like they could ask just people on the Internet how many people use Internet. Everybody would say yes and it would look like everybody in the world uses Internet."

Ken suggested that they start a class list of questions to consider when reading and interpreting charts and graphs. The first one they wanted him to write was to "ask yourself what types of people were asked the questions." Ken wrote it down and asked whether they should consider how all the data used in graphs were collected, not just for surveys with people. They added that to their list.

After completing their survey, students used it to interview other adult learners in their adult education program. Ken then collected all the results, asked the students to break into small groups, made multiple copies so that each group had the same set of data, and asked the groups to

organize and then analyze the data. Ultimately, they needed to prepare a report and an oral presentation describing the results of their survey. He required that students, in their report, include implications based on the data. He did not just want them reporting on the data, but also what the data suggested. Ken wanted his students to be critical thinkers, not just spouters of facts.

Ken and the students used this presentation as an assessment of what they learned about collecting and analyzing data. It helped them decide what they needed to learn next. Ken observed and listened to students to see the types of graphs they constructed. Based on his observations and students' feedback, he decided to focus more closely on pie charts next.

What a Deal!

On several occasions, Ruben has overheard his students talking about rent-to-own "deals". He wonders whether they are comfortable making informed decisions about what plan to choose so he asks them. He discovers that his students do not know how to figure out whether to rent-to-own, buy on layaway, use a credit card, or try to save until they have enough in savings,. He asks whether they would like to learn more about how to make informed decisions about choosing a plan that works for them. He offers to also use this as an opportunity to show how applicable algebra is to real-life and to build on their work with equations.

He asks students to bring in a variety of examples of rent-to-own offers. He also asks them to research similar products that could be bought at department and discount stores and other ways to buy big-ticket items. When they have provided him with enough examples, he models how algebraic equations work to help adults make comparisons between and among various situations.

He begins with the situation – a department store offers customers the following deal on furniture for the home: Put 10% down, then pay monthly payments for 3 years. Students decide that they would like to see how much a television would cost. After some discussion, they settled on a \$599 TV that offered 3 years of payments at \$20.35 per month.

He helps the students transform the situation into an equation by writing out the following as the students dictate to him:

Total cost of the TV = 10% of \$599 + 3(\$20.35). They suggest he round \$599 to \$600 for the purpose of the group modeling, so he changes \$599 to \$600. Since they have had lots of exposure in earlier work to benchmark percents, they can calculate 10% of 600 mentally.

They suggest that the equation now should read:

Total Cost = $60 + 3(20.35)$. Several students who are mentally rounding 20.35 to 20 and multiplying 3×20 comment that something must be wrong because the total is much less than the advertised cost of \$599. Together they ponder out loud how their equation could be wrong while Ruben quietly listens.

Finally, they discover their error and a student comes to the board and writes:

$C = 10\% (\$600) + 36(\$20.35)$. She tells Ruben that the error was, “We used 3 years but we need to know the number of months, so we need to change it to 36 instead.”

After students calculate 36 times 20.35, the student writes on the board:

$$C = 60 + \$732.60$$

$$C = \$792.60$$

Before assigning the students in groups to work on the other examples that they brought in, Ruben gives them some guidelines. They are to come up with the total cost of the item once it is finally paid for. They are also to develop a rule or formula for figuring out the total cost. He feels that being able to come up with ways to figure the total cost over time will help students be able to make informed decisions when they decide to make large purchases.

When groups are done, students share their findings. They are astounded to realize the total cost of items that were bought through the rent-to-own method. They talk at length about the importance of being comfortable in using math so that they can make decisions based on knowledge rather than persuasion.

WA Level 6: Use Math to Solve Problems and Communicate

WA Level 6 Indicators

M 6.1 Read, write, and interpret a wide variety of (often) complex mathematical information such as

Numbers and number sense: money/expenses/pricing.

Patterns/Functions/Relationships: formulas for a variety of calculations.

Space/Shape/Measurement: architectural symbols/models and scale modeling.

Data/Statistics: ways to interpret, represent, identify trends in or draw inferences from data (complex tables and graphs; advanced forms of statistical analysis; graphing equations and generating equations from data and/or line graphs; using concept of slope).

M 6.2 Research, select and apply sophisticated, multi-step mathematical concepts and procedures (such as scale modeling, cost analysis, earnings/deductions analysis).

M 6.3 Evaluate the degree of precision needed for the solution.

M 6.4 Independently research, select, organize and integrate mathematical information of different types in carrying out procedures, describing patterns, and/or measuring with appropriate tools, to solve the problem and to verify that the solution is reasonable.

M 6.5 Create appropriate visual or graphic representations such as charts, tables, graphs, etc. and clearly communicate the solution process and results orally or in writing to a variety of audiences.

Show Fluency, Independence and Ability to Perform in a Range of Settings

Adults performing at Level 6 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish minimally structured, complex math tasks in a variety of comfortable and familiar settings.

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables and algebraic models.

Teaching and Learning Objectives

To support performance of the Standard at this level, instruction should be offered in the context of real-life purposes, texts, and activities, and should, across the four strands, address the integration of knowledge, skills, and strategies such as those below:

Number Sense Strand

6A1 Read, write, and interpret a wide variety of (often) complex mathematical information:

- a. Recognize and use positive and negative numbers of any size
- b. Read, write, and compare fractions and mixed numbers and decimals

6A2 Research, select, and apply sophisticated, multi-step mathematical concepts and procedures:

- a. Demonstrate an understanding that the associative, distributive, and commutative properties are used to make problem solving simpler

6A3 Determine degree of precision

6A4 Use computational procedures effectively to solve real-life problems:

- a. Add, subtract, multiply, divide with decimals of any size
- b. Add, subtract, multiply, divide using fractions of any size
- c. Add subtract, multiply, divide integers
- d. Apply percent for proportion
- e.

6A5 Communicate results, including the solution process, orally, in writing, in tables, graphs, etc.

Patterns, Functions, Relationships Strand

6B1 Read, write, and interpret a wide variety of (often) complex patterns, functions and relationships:

- a. Develop and apply formulas for a variety of calculations
- b. Create and analyze a wide variety of patterns, including quadratic, linear, and exponential functions
- c. Explain the difference between linear and exponential growth
- d. Convert between different representations (formulas vs. tables, vs. graphs, etc.)
- e. Describe general shape and qualities of linear and simple non-linear graphs
- f. Demonstrate an understanding that different representations can be used to illustrate a pattern, function, or relationship

6B2 Read and interpret symbolic information:

- a. Recognize and apply exponential notation
- b. Evaluate formulas and functions
- c. Solve equations, including linear, quadratic, and exponential

Space/Shape/Masurement Strand

6C1 Read, write, and interpret a wide variety of (often) complex mathematical information related to measurement:

- a. Know vocabulary for trigonometric functions
- b. Read trig tables or use scientific calculator to find trig ratios
- c. Use coordinates to describe rotations or changes of geometric figures

6C2 Research, select, and apply sophisticated, multi-step mathematical concepts:

- a. Solve problems involving perpendicularity, parallelism, congruence and similarity of geometric figures (including polygons and three-dimensional figures)
- b. Analyze situations involving cost (such as profit/loss margin) or earnings/deductions

Data and Statistics Strand

6D1 Read and interpret data and statistical information:

- a. Make absolute comparisons about relative values on graphs and tables
- b. Infer meaning from gaps, clusters, trends, and comparisons of data
- c. Predict consequences related to data interpretations
- d. Identify the shape, range, median, mean, and mode of data
- e. Describe the effect of range on the mean and median and which is most appropriate for the data
- f. Make statements using interpretations from data
- g. Demonstrate an understanding that sample size and representation, scale, and categorization of data can distort interpretations

WA Level 6 Examples of Proficient Performance

Adults performing at Level 6 can select, analyze, integrate, and use mathematical information to accomplish a variety of goals, such as:

- Analyze effects of deductions on earnings and projecting annual income
- Design a “dream house”
- Use an amortization table to decide whether to refinance the mortgage on a house
- Determine a budget for a grant proposal
- Design an archway or bridge to scale
- Make a decision about how to consolidate bills and credit card payments

6D2 Collect, organize, and represent data

- a. Design research question, then collect and organize data to answer question
- b. Choose appropriate representations to display data collected
- c. Collect comparative data on a single question (such as responses grouped by gender vs. by ethnicity)
- d. Display comparative data on a double bar or line graph

6D3 Recall and use basic probability concepts:

- a. State probability in different formats: ratio fraction and percent
- b. Find the probability of independent and dependent events

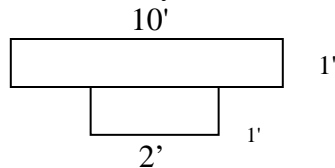
Examples of a Person Performing at WA Level 6:

Pond Ponderings

Donna had dug an irregularly shaped pond and now needed to add clarifier to get rid of the algae. The back of the clarifier suggested adding 50 mL per 265 gallons of water. Since she had dug the pond free-form fashion to fit the area of her yard, she did not know how many gallons of water the pond held.

However, Donna knew that gallons referred to volume – the amount of water that the pond could hold. She knew that volume was three-dimensional, so she realized that if she could estimate the length, width, and depth of her pond, she could calculate its volume. Since she measured her pond in terms of linear feet, she knew this would give her the dimensions in terms of cubic feet, not gallons but she knew this was not a problem – all she had to do was look up (in a dictionary, a math book, or even the Internet) to find the number of cubic feet in a gallon.

Even though the pond was a somewhat oval shape, she mentally changed it to a rectangle for ease of manipulating numbers (because she also knew that she did not have to be exact in her answer). The center section of the pond was two feet deep, so she roughly estimated the shape to be 2 feet deep, by 7 feet long and 3 feet wide. She then noted that the rest of the pond was about a foot deep and was about 10 feet long and 6 feet wide. She decided to think about the pond as if it were in two layers. She readjusted her estimated of the deep section to be 1 x 7 x 3 since that would be considered one layer while the next layer (one foot deep) spanned the entire pond.



She multiplied the two separate sections to figure the cubic feet of each ($1 \times 7 \times 3$ and $1 \times 10 \times 6$), then added the two sets of cubic feet together: $21 + 60 = 81$ cubic feet. She then went online and found out that 1 gallon = 0.133680556 cubic feet. She decided to round to .134 and then used the cross product strategy since the numbers were not very friendly:

$$\frac{1 \text{ gal.}}{.134 \text{ cu. ft}} = \frac{x \text{ gal.}}{81 \text{ cu. ft}}$$

She discovered that her pond held about 604 gallons. This sounded reasonable once she checked her estimate using proportional thinking:

$$\begin{aligned} 1 \text{ gal} &= .134 \text{ cu. ft} \\ 100 \text{ gal} &= 13.4 \text{ cu. ft} \\ (6 \times 100 \text{ gal}) \quad 600 \text{ gal} &= 78 \text{ cu. ft} \quad (6 \times 13 \text{ cu. ft}) \end{aligned}$$

She then used the cross product method again to determine the number of mL of clarifier she needed:

$$\frac{50 \text{ mL}}{265 \text{ gal.}} = \frac{x \text{ mL}}{604 \text{ gal.}}$$

She figured she needed 114 mL of clarifier for her pond.

Keeping a Monthly Budget

Deborrah finds that she is always short of money by the end of the month even though she thinks she has planned ahead. She decides that she needs a strategy to help her plan more effectively. Some of her friends have mentioned that they keep track of their expenses using a spreadsheet which allow them to keep better track of all their expenses – and more accurately since they rely on the computer to do the calculations once the expenses are inputted.

Deborrah has had some limited experience with using a spreadsheet and she is comfortable enough with using basic algebraic equations, so she thinks she can figure out what she needs. She first determines what her known monthly expenses are and inputs those into the spreadsheet – things like rent and car payment. She knows that she has other expenses that occur every month, but they vary – things like utilities, food, and gas for transportation.

She decides to get her past year's utility bills and averages them to get an estimated monthly cost. She notes that the bills are higher during some months than others; she decides to do a monthly average and then monitor to see whether it makes a difference in her budget planning. She really has not paid attention to how much she pays out per month for food – whether it is for groceries or eating out with friends. She suspects that this might be part of the problem with her end of month budget woes.

She knows what her monthly income is so knows about the total amount that she can spend per month. She does a rough estimate of what she thinks she has been paying on average for groceries, gas, and other monthly needs. She figures she will need to compare her estimate to actual expenses over the next few months in order to see whether her predictions are correct, and whether she will need to make adjustments in her spending habits in order to meet her budgetary goals.

Examples of Teaching and Learning at WA Level 6:

(Exit point for National Reporting System High Adult Secondary Education)

Exponential Growth

Diana was working with a group of students who were interested in taking the placement test for college. She knew that one of the areas that would be tested would be various equations, including both linear (those forming a straight line when graphed) and quadratic (involving a number squared). From the past work she had been doing with these students, she knew that they were becoming comfortable with linear equations and had several experiences solving them as well as creating graphs.

To introduce the graph of a quadratic equation, she decided to begin with a real-life situation building on what her students were already familiar with. A few days earlier, she had heard a couple of her students talking about building a patio. She thought she might use this to initiate a discussion about different types of equations. Students were excited about the prospect of learning how to design and cost out their own patios, especially knowing that the concepts that they would be learning would prepare them for the placement test as well.

She had them each first create their own patios. She suggested that they begin with a small square shape and design it on graph paper. She then asked them to determine both the area and the perimeter of the square they had created. Since they were familiar with solving problems using formulas, most of the students used the formulae for area and perimeter. A few chose to simply count the number of squares to figure the area, and added all four sides to determine the perimeter.

Diana then asked them to predict what would happen if they doubled the sides of their square. Several predicted that both the perimeter and area would double. She asked them to test out their predictions; they discovered that the perimeter did indeed double. They also discovered that the area was much larger than they had anticipated. She suggested that groups of students compare their squares and resulting perimeters and areas since this would give them a chance to look at several different sizes of shapes.

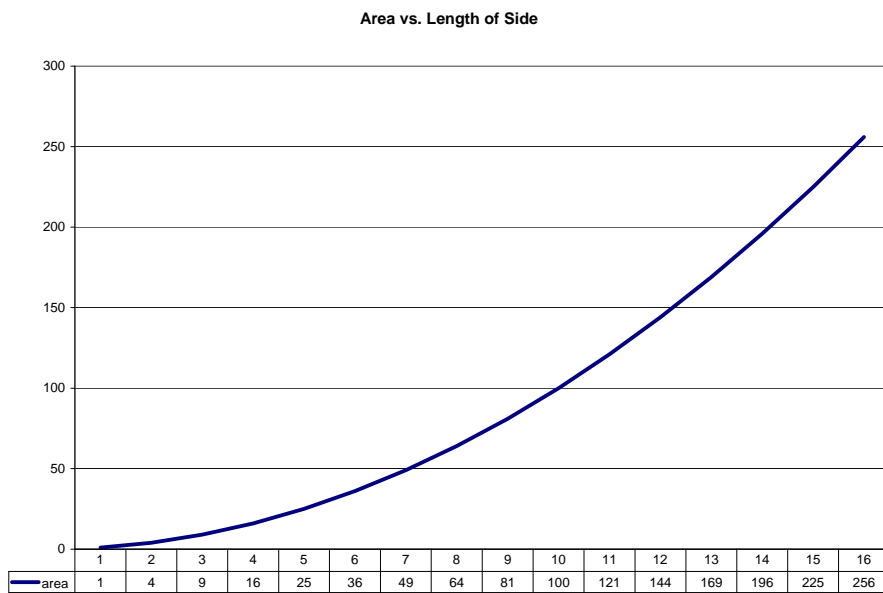
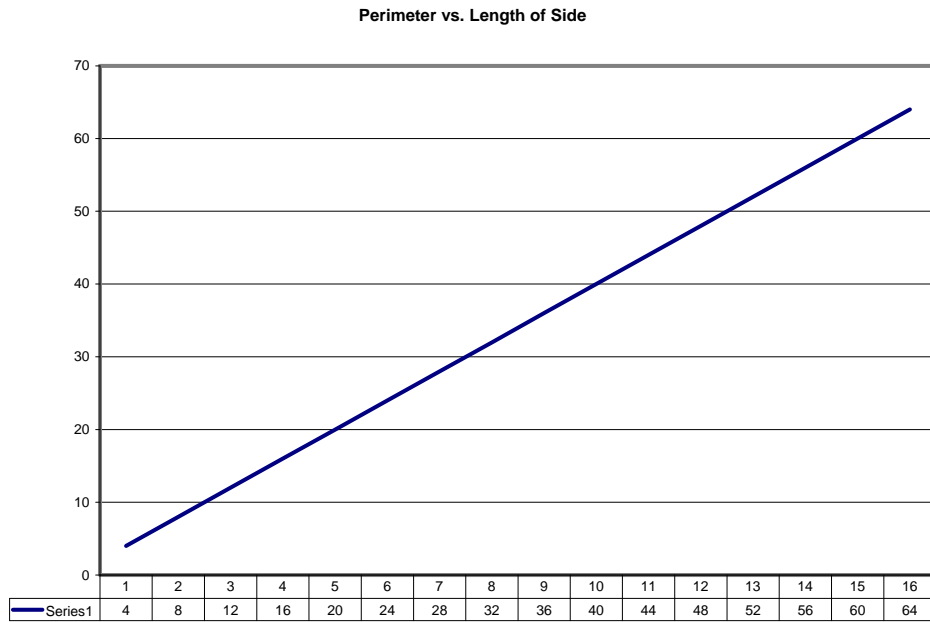
She encouraged students to create a table of perimeters and areas to see if they could see a pattern. She modeled by asking for volunteers to share the size of their squares. One group started with a 10 x 10 square. As they shared their information, Diana wrote on the board:

Length of side	Perimeter	Area
10 feet	40 feet	100 sq. ft.
20 feet	80 feet	400 sq. ft.

She then asked for other sizes of shapes and suggested that it would be easier to look for patterns if the number were in order. She asked for suggestions on how to organize the data and the students decided that the size of the square might work. She added all their examples to the chart in order of square size. She then suggested the create their own table, beginning with a square with a side the length of one foot, a side with a length of two feet, and a side with a length of four feet, as well as one other small size square.

She then asked students to look at the patterns that they saw occurring. She nudged them in their thinking so that they could see that the area kept increasing by perfect squares (1, 4, 9, 25, etc.) while the perimeter was always four times the length of the side of the square.

Students then worked in pairs to graph each of the sets of data (at least the smaller numbers) after predicting what the shape of the line would be. The resulting graphs led to much further discussion about exponential growth, including examples of where such growth might occur.



Students also talked about the shapes of the patios and the relationship between the perimeter and the area. They discussed how costly a patio could become because the area of the patio was what

would have to be filled with bricks or stone. They noted that, each time they decided to make their square patio even one brick wider and longer, the cost would increase considerably. “It’s not the same as thinking that you would just have to add on the cost of one row of bricks to the price,” stated one student. “You have to realize that you have to add bricks to both the length and the width in order to keep the square shape – and that would mean having to pay more because the area is growing so big.”

Your Version of the Story

Several of the students in Mary Ellen’s math class were talking about some statistics that they had recently heard about regarding a recent election. They were discussing whether the statistics were actually true or not. They asked Mary Ellen what she thought.

She realized that her students were not familiar with how some individuals use statistics to provide their own “version” of the data. She decided to work with the students to create an activity that would illustrate how data can be manipulated to give varying perspectives.

The students were to collect data, organize them, and present the results in at least two different ways. In presenting the data, they were to consider who might want to present data in such a way as to influence others. Since students had never considered that someone might want to present data in order to influence (they thought that “facts were facts”), she showed them the following example:

In a company of 100 employees, 35% are women. The company decided to cut back on benefits for part-time workers. Many women complained because they felt they were being discriminated against. The company president said that could not be so since there are many more male workers than female workers affected. He reported that there were 26 men who would be impacted but only 21 women.” The company president was using *absolute numbers* – the actual difference between the number of men and women who are part-time.

The women went to the union who understood their concern. Sixty percent of the women employees are part-time, while only 40% of the men are part-time.” Therefore, proportionally, there are more women than men who are part-time employees. The union was using percents, which are considered relative numbers - numbers that show a relationship between two things.

They also talked about using measures of central tendency (sometimes called “averages”) in describing sets of data. They were already familiar with mean, median, and mode, but had not really looked at how they could be used to influence. So, Mary Ellen asked them to figure out the mean, median, and mode of the following simple salaries:

Boss: \$100,000

Boss’ son: \$50,000

Two workers: \$25,000

Students readily calculated that the mean was \$50,000, the median was \$37,500, and the mode was \$25,000. She then asked them to consider who would most likely benefit from talking about the “average” salary of the company being \$50,000. Who would most likely benefit from saying that the “average” salary was about \$25,000? Students began to see how choosing to use different statistics or describing situations in terms of relative vs. absolute numbers could influence how the data are interpreted.

Mary Ellen, after these lessons, felt that students were now ready to try to collect their own data to organize, interpret, and present from at least two different perspectives.

WA Use Math to Solve Problems and Communicate Table of Teaching and Learning Objectives

A. NUMBER SENSE STRAND

WA Levels 1 & 2	WA Level 3	WA Level 4	WA Level 5	WA Level 6
<p>1&2A1 Read, write, and interpret very simple types of mathematical information:</p> <ul style="list-style-type: none"> a. Read, write, and compare whole numbers up to three digits b. Understand and use place value to hundreds c. Count by 2s, 5s, 10s d. Recognize common monetary values e. Demonstrate an understanding that 50% is the same as $\frac{1}{2}$ and 25% is the same as $\frac{1}{4}$ 	<p>3A1 Read, write, and interpret common types of mathematical information:</p> <ul style="list-style-type: none"> a. Read, write, and compare whole numbers up to 10,000 b. Read, write, and compare whole numbers and benchmark fractions ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{10}$), decimals, and percents c. Recognize and use equivalent forms of benchmark fractions (such as $\frac{2}{4} = \frac{1}{2}$) d. Understand and use place value (to the nearest hundredth as in monetary values) e. Demonstrate and understanding of and recognize and apply simple negative numbers in real contexts (such as thermometers) 	<p>4A1 Read, write, and interpret a variety of common mathematical information:</p> <ul style="list-style-type: none"> a. Read, write, and compare whole numbers (to billions) b. Read, write, and compare extensions of benchmark fractions, decimals, and percents (such as $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{100}$) c. Convert extensions of benchmark fractions, decimals, and percents to their equivalents d. Recognize and apply negative numbers in real contexts (such as thermometers, winning and losing money) e. Read, write, and compare decimals to three places 	<p>5A1 Read, write, and interpret a wide variety of mathematical information:</p> <ul style="list-style-type: none"> a. Recognize and use positive and negative numbers of any size b. Read, write, and compare fractions and mixed numbers and decimals 	<p>6A1 Read, write, and interpret a wide variety of (often) complex mathematical information:</p> <ul style="list-style-type: none"> a. Recognize and use positive and negative numbers of any size b. Read, write, and compare fractions and mixed numbers and decimals

<p>1&2A2 Recall and use a few simple procedures:</p> <ul style="list-style-type: none"> a. Estimate with numbers up to 3 digits b. Sort, order, group by color, shape, etc. c. Add-on using counting or calculator d. Subtract, including the understanding that addition and subtraction are related e. Demonstrate an understanding of the multiplication facts up to 12 (such as repeated addition, a combination of multiplication and adding on, etc.) f. Demonstrate an understanding that add and multiply mean to combine, subtract means to take away, and divide means to separate into groups 	<p>3A2 Recall and use mathematical procedures:</p> <ul style="list-style-type: none"> a. Estimate with whole numbers b. Group, compare c. Add and subtract (with or without calculator) d. Demonstrate an understanding that multiplying by a simple fraction is the same as dividing by the whole number in the denominator (such as $1/2 \times 10$ is the same as $10 \div 2$) e. Demonstrate an understanding that finding the root and square root are related 	<p>4A2 Recall and use a good store of mathematical procedures:</p> <ul style="list-style-type: none"> a. Estimate with benchmark fractions, decimals, and percents b. Multiply and divide accurately (with and without calculator) c. Add and subtract common fractional amounts d. Calculate percent increase or decrease with friendly numbers 	<p>5A2 Recall and use multi-step mathematical procedures:</p> <ul style="list-style-type: none"> a. Demonstrate an understanding of what happens when fractions are added, subtracted, multiplied, or divided b. Demonstrate an understanding of what happens when decimals are added, subtracted, multiplied, or divided c. Demonstrate an understanding of what happens when integers are added, subtracted, multiplied, or divided d. Calculate percent increase or decrease 	<p>6A2 Research, select, and apply sophisticated, multi-step mathematical concepts and procedures:</p> <ul style="list-style-type: none"> a. Demonstrate an understanding that the associative, distributive, and commutative properties are used to make problem solving simpler
<p>1&2A3 Determine degree of precision, rounding to the nearest 10, 100</p>	<p>3A3 Determine degree of precision, rounding to the nearest 10, 100, 1000, 10,000</p>	<p>4A3 Determine degree of precision (such as to the nearest billion or to the nearest tenth or hundredth)</p>	<p>5A3 Determine degree of precision</p>	<p>6A3 Determine degree of precision</p>

<p>1&2A4 Use computational procedures effectively to solve real-life problems:</p> <ul style="list-style-type: none"> a. Accurately adds and subtracts one and two digit numbers using manipulatives, grouping, or calculator b. Determine one-half or one-quarter of an amount using manipulatives, drawings, or other simple strategies c. Determine 50% or 25% of an amount d. Can double very simple whole number amounts e. Can apply simple proportions such as 2:1, 1:2 f. Use calculator and drawings to check for reasonableness of answer 	<p>3A4 Use computational procedures effectively to solve real-life problems:</p> <ul style="list-style-type: none"> a. Accurately add and subtract whole numbers b. Multiply and divide three digit whole numbers using effective strategies c. Determine parts of a whole using benchmark fractions such as $\frac{3}{4}$ and $\frac{1}{10}$ d. Understand and use simple strategies for determining benchmark decimals (.25, .50, .75, .10) e. Understand and use simple strategies for determining benchmark percents (25%, 50%, 75%, 10%) f. Articulate the relationship between benchmark fractions, decimals, and percents (such as .50, 50%, $\frac{1}{2}$) g. Can apply proportions such as 4:1, 1:4 h. Check for reasonableness of answer using estimation and benchmarks 	<p>4A4 Use computational procedures effectively to solve real-life problems:</p> <ul style="list-style-type: none"> a. Use extensions of benchmark fractions ($\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{5}$, etc.), decimals, and percents (15%, 30%, etc.) b. Add, subtract, multiply, divide with any whole numbers using effective strategies c. Add and subtract decimals up to three places d. Use and demonstrate understanding of order of operations in multi-step problems e. Use benchmark fractions decimals and percents to check for reasonableness of answer 	<p>5A4 Use computational procedures effectively to solve real-life problems:</p> <ul style="list-style-type: none"> a. Add, subtract, multiply, divide with decimals b. Add, subtract, multiply, divide using fractions c. Compute with percentages 	<p>6A4 Use computational procedures effectively to solve real-life problems:</p> <ul style="list-style-type: none"> a. Add, subtract, multiply, divide with decimals of any size b. Add, subtract, multiply, divide using fractions of any size c. Add subtract, multiply, divide integers d. Apply percent for proportion
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<p>1&2A5 Communicate results in role plays, with pictures, and by entries on a simple chart</p>	<p>3A5 Communicate results orally, in pictures, entries in a chart, and in writing</p>	<p>4A5 Communicate results orally, in pictures, entries in a table or graph, with basic statistics (range, mean, median, mode), and in writing</p>	<p>5A5 Communicate results, including the solution process, orally, in writing, in tables, graphs, etc.</p>	<p>6A5 Communicate results, including the solution process, orally, in writing, in tables, graphs, etc.</p>
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B. PATTERNS/ FUNCTIONS/ RELATIONSHIPS STRAND

WA Levels 1 & 2	WA Level 3	WA Level 4	WA Level 5	WA Level 6
<p>1&2B1 Read, write, and interpret very simple patterns, functions and relationships:</p> <p>a. Recognize and complete number patterns up to 1000 and generalize the relationship</p> <p>b. Create tables for addition and multiplication</p>	<p>3B1 Read, write, and interpret simple patterns, functions and relationships:</p> <p>a. Recognize, describe, and generalize for a rule for patterns</p> <p>b. Recognize and develop repeating patterns and generalize the relationship</p> <p>c. Generalize the relationship between quantities in a table of amounts</p> <p>d. Create simple expressions or formulas from real-life situations</p> <p>e. Create and apply very simple formulas from tables with one arithmetic step</p>	<p>4B1 Read, write, and interpret a variety of common patterns, functions and relationships:</p> <p>a. Recognize and describe patterns and apply simple formulas</p> <p>b. Recognize and develop repeating patterns and generalize the relationship with a table, rule, or simple formula</p> <p>c. Create simple expressions or formulas from real-life situations</p> <p>d. Create and apply simple formulas from tables and create simple linear graphs</p>	<p>5B1 Read, write, and interpret a wide variety of patterns, functions and relationships:</p> <p>a. Recognize, describe, and extend patterns and apply formulas</p> <p>b. Demonstrate an understanding that patterns and relationships can be represented with a formula, a table, or a graph</p> <p>c. Develop algebraic expressions, generalizations, or formula for simple number patterns</p> <p>d. Describe general shape and qualities of linear and simple non-linear graphs</p>	<p>6B1 Read, write, and interpret a wide variety of (often) complex patterns, functions and relationships:</p> <p>a. Develop and apply formulas for a variety of calculations</p> <p>b. Create and analyze a wide variety of patterns, including quadratic, linear, and exponential functions</p> <p>c. Explain the difference between linear and exponential growth</p> <p>d. Convert between different representations (formulas vs. tables, vs. graphs, etc.)</p> <p>e. Describe general shape and qualities of linear and simple non-linear graphs</p> <p>f. Demonstrate an understanding that different representations can be used to illustrate a pattern, function, or relationship</p>

<p>1&2B2 Read and interpret symbolic information:</p> <ul style="list-style-type: none"> a. Recognize and name all ten digits b. Recognize and name +, -, = symbols c. Can solve simple number sentences such as $3 + \square = 5$ d. Orally describe number relationships such as 3 + 2 does not equal 7. e. Read positive and negative numbers 	<p>3B2 Read and interpret symbolic information:</p> <ul style="list-style-type: none"> a. Use exponents to show repeated multiplication fro simple amounts (such as $3^2 = 9$) b. Read and write operations using algebraic notation (parentheses or \cdot for multiplication and $/$ for division) c. Evaluate one-step equations 	<p>4B2 Read and interpret symbolic information:</p> <ul style="list-style-type: none"> a. Use exponents to show repeated multiplication in real contexts b. Evaluate simple formulas and expressions c. Apply order of operations to solve equations and expressions d. Write statements of equality and inequality (such as $3 > 4 - 3$) 	<p>5B2 Read and interpret symbolic information:</p> <ul style="list-style-type: none"> a. Solve problems in two-step equations b. Evaluate formulas c. Solve linear equations 	<p>6B2 Read and interpret symbolic information:</p> <ul style="list-style-type: none"> a. Recognize and apply exponential notation b. Evaluate formulas and functions c. Solve equations, including linear, quadratic, and exponential
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C. SPACE/ SHAPE/ MEASUREMENT STRAND

WA Levels 1 & 2	WA Level 3	WA Level 4	WA Level 5	WA Level 6
<p>1&2C1 Read, write, and interpret very simple types of information related to measurement:</p> <ul style="list-style-type: none"> a. Read and write time (hours and minutes, day of month, etc.) b. Describe measurable attributes such as weight and height by comparing (one is heavier than the other) c. Read measuring tools (rulers, measuring cups) to nearest whole unit) d. Recognize and describe familiar geometric shapes (such as circle, square, triangle, etc.) e. Describe a shape based on dimensions of length and width f. Show equivalent amounts of money using different coins and dollar bills g. Use coordinate geometry to specify locations and relationships (such as east and west, north and south) on a map 	<p>3C1 Read, write, and interpret simple types of information related to measurement:</p> <ul style="list-style-type: none"> a. Recognize and use commonly used standard units of measurement to the nearest half b. Recognize and describe common two- and three-dimensional shapes, including angle descriptions c. Identify right angles d. Describe informally the difference between square units (such as square inches) and linear units (such as inches) e. Use direction, distance, labels, simple scales, and symbols to read and use maps and plans f. Read and compare positive and negative temperatures on a Fahrenheit thermometer 	<p>4C1 Read, write, and interpret a variety of common mathematical information related to measurement:</p> <ul style="list-style-type: none"> a. Measure and compare radius, diameter, circumference of a circle b. Use a protractor to measure angles c. Use scale to find distance between locations on a map d. Locate points on a globe or map using latitude and longitude 	<p>5C1 Read, write, and interpret a wide variety of mathematical information related to measurement:</p> <ul style="list-style-type: none"> a. Use the language of metric unit prefixes to describe real-life measurements b. Demonstrate an understanding of the coordinate graph system, including ordered pairs 	<p>6C1 Read, write, and interpret a wide variety of (often) complex mathematical information related to measurement:</p> <ul style="list-style-type: none"> a. Know vocabulary for trigonometric functions b. Read trig tables or use scientific calculator to find trig ratios c. Use coordinates to describe rotations or changes of geometric figures

<p>1&2C2 Recall and use a few simple mathematical procedures:</p> <ul style="list-style-type: none"> a. Measure involving high frequency standard whole units of measure such as pounds, inches, feet, quarts, gallons, cups b. Use appropriate tools to measure to the nearest whole unit c. Find the area and perimeter of rectangles using simple strategies such as drawings or adding on d. Calculate the total cost of several items and the amount of change due (for amounts up to \$20.00) 	<p>3C2 Recall and use mathematical procedures:</p> <ul style="list-style-type: none"> a. Solve problems involving money, including decimals b. Measure with a ruler to the nearest half and quarter c. Use appropriate tools to measure to the nearest half unit d. Demonstrate an informal understanding of the difference between inch and centimeter, yard and meter e. Make simple conversions within the same measurement system (such as inches to feet, cm to m) 	<p>4C2 Recall and use a good store of mathematical procedures:</p> <ul style="list-style-type: none"> a. Convert units of measure from one system to another using informal methods b. Use appropriate tools to measure to the nearest hundredth c. Find the volume of cubes 	<p>5C2 Recall and use multi-step mathematical procedures:</p> <ul style="list-style-type: none"> a. Use ratio and proportion in familiar settings involving scale drawings or maps b. Develop and apply formulas for perimeter, area, and volume c. Apply the knowledge of 180 degrees in a triangle to solve problems d. Use the Pythagorean theorem to problems involving right triangles e. Determine the slope of a line, the y-intercept, and the intersection of two lines f. Predict the impact of changes in linear dimensions on the perimeter, area, and volume of a figure 	<p>6C2 Research, select, and apply sophisticated, multi-step mathematical concepts:</p> <ul style="list-style-type: none"> a. Solve problems involving perpendicularity, parallelism, congruence and similarity of geometric figures (including polygons and three-dimensional figures) b. Analyze situations involving cost (such as profit/loss margin) or earnings/deductions
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D. DATA/ STATISTICS STRAND

WA Levels 1 & 2	WA Level 3	WA Level 4	WA Level 5	WA Level 6
<p>1&2D1 Read and interpret data and statistical information:</p> <p>a. Extract discrete information from simple and concrete graphs, or from a list or table</p> <p>b. Describe concrete information found in a graphic representation</p> <p>c. Make numerical comparisons about relative values on a bar graph (such as “greater than” or “less than” or about twice as much”)</p> <p>d. Use titles and labels to make statements about data representations</p>	<p>3D1 Read and interpret data and statistical information:</p> <p>a. Extract discrete information from a list, table, or bar graph</p> <p>b. Make numerical comparisons about relative values on a bar graph (such as “greater than” or “less than” or “about twice as much”)</p> <p>c. Identify the shape, range, median, and mean of data</p> <p>d. Make statements and simple comparisons based on data representations</p> <p>e. Demonstrate an understanding that bar widths and scale can distort interpretations of data</p>	<p>4D1 Read and interpret data and statistical information:</p> <p>a. Make absolute comparisons about relative values on graphs and tables</p> <p>b. Identify the shape, range, mean, median, and mode of data</p> <p>c. Describe the effect of range on median and mean and which is most appropriate for the data</p> <p>d. Make simple comparisons to support or refute statements based on representations of data</p> <p>e. Demonstrate an understanding that scale and sample can distort interpretations of data</p>	<p>5D1 Read and interpret data and statistical information:</p> <p>a. Make absolute comparisons about relative values on graphs and tables</p> <p>b. Infer meaning from gaps, clusters, trends, and comparisons of data</p> <p>c. Predict consequences related to data interpretations</p> <p>d. Identify the shape, range, median, mean, and mode of data</p> <p>e. Describe the effect of range on the mean and median and which is most appropriate for the data</p> <p>f. Make statements using interpretations from data</p> <p>g. Demonstrate an understanding that sample size and representation, scale, and categorization of data can distort interpretations</p>	<p>6D1 Read and interpret data and statistical information:</p> <p>a. Make absolute comparisons about relative values on graphs and tables</p> <p>b. Infer meaning from gaps, clusters, trends, and comparisons of data</p> <p>c. Predict consequences related to data interpretations</p> <p>d. Identify the shape, range, median, mean, and mode of data</p> <p>e. Describe the effect of range on the mean and median and which is most appropriate for the data</p> <p>f. Make statements using interpretations from data</p> <p>g. Demonstrate an understanding that sample size and representation, scale, and categorization of data can distort interpretations</p>

<p>1&2D2 Collect, organize, and represent data:</p> <ul style="list-style-type: none"> a. Collect and organize data based on posed question and organize in a reasonable manner b. Represent data in simple forms such as check sheets, picture graphs, simple bar graphs, and line plots c. Use simple charts or tables to communicate information d. Verify that data represented is the actual data collected 	<p>3D2 Collect, organize, and represent data:</p> <ul style="list-style-type: none"> a. Collect and organize data based on posed question and organize in a reasonable manner b. Display categorical data in a simple bar graph or line plot c. Verify that data represented is the actual data collected 	<p>4D2 Collect, organize, and represent data:</p> <ul style="list-style-type: none"> a. Collect and organize data based on posed question and represent the information in a table, list, line plot, or frequency table b. Display categorical data in a bar graph or circle graphs c. Convert bar graphs into circle graphs 	<p>5D2 Collect, organize, and represent data:</p> <ul style="list-style-type: none"> a. Collect and organize data based on posed questions b. Choose appropriate representations to display data collected c. Collect comparative data on a single question (such as responses grouped by gender vs. by ethnicity) d. Display comparative data on a double bar or line graph 	<p>6D2 Collect, organize, and represent data:</p> <ul style="list-style-type: none"> a. Design research question, then collect and organize data to answer question b. Choose appropriate representations to display data collected c. Collect comparative data on a single question (such as responses grouped by gender vs. by ethnicity) d. Display comparative data on a double bar or line graph
<p>1&2D3 Recall and use basic probability concepts:</p> <ul style="list-style-type: none"> a. Describe the probability of an event within the range of “possible” or “impossible” b. Find the probability of a single outcome in a simple concrete situation such as a coin toss or die roll 	<p>3D3 Recall and use basic probability concepts:</p> <ul style="list-style-type: none"> a. State probability in a ratio fraction b. Find the probability of a single outcome in a simple concrete situation with a very limited number of possible outcomes 	<p>4D3 Recall and use basic probability concepts:</p> <ul style="list-style-type: none"> a. State probability in ratio fraction and compare to benchmark fractions b. Find the probability of a single outcome in a concrete situation 	<p>5D3 Recall and use basic probability concepts:</p> <ul style="list-style-type: none"> a. State probability in different formats: ratio fraction and percent b. Find the probability of independent and dependent events 	<p>6D3 Recall and use basic probability concepts:</p> <ul style="list-style-type: none"> a. State probability in different formats: ratio fraction and percent b. Find the probability of independent and dependent events

APPENDIX A: WA, EFF, CASAS and NRS Levels

Correspondences among WA Levels, EFF Levels and NRS, (ABE, ESL, and MATH) Educational Functioning Levels

EFF Levels	WA ESL Levels	NRS Federal ESL Levels	CASAS Scores for ESL	WA ABE Levels	NRS Federal ABE & MATH Levels	CASAS Scores: ABE & MATH
Beginning ESL Literacy	1	Beginning ESL Literacy	180 and below			
Beginning ESL	2	Low Beginning ESL	181-190			
1	3	High Beginning ESL	191-200	1	Beginning ABE Literacy	200 and below
2	4	Low Intermediate ESL	201-210	2	Beginning Basic Education	201-210
3	5	High Intermediate ESL	211-220	3	Low Intermediate Basic Education	211-220
4	6	Advanced ESL	221-235	4	High Intermediate Basic Education	221-235
5				5	Low Adult Secondary Education	236-245
6				6	High Adult Secondary Education	246+

For information about the NRS Levels, go to: http://www.nrsweb.org/docs/EFL_Discriptors.doc
 Adapted from: The Equipped for the Future Read With Understanding Curriculum Framework; Appendix A, 2006 Edition.

Glossary

Algebra: The use of numbers, letters, and other systematized symbols to express and analyze the relationship between concepts of quantity in terms of formulas, equations, etc., including generalized arithmetic.

Associative property of addition: Rule which says: When adding three or more numbers, it does not matter whether the first pair or the last pair is added first; for example $2 + 3 + 8 = (2 + 3) + 8$ and $2 + (3 + 8)$.

Associative property of multiplication: Rule which says: When multiplying three or more numbers, it does not matter whether the first pair or the last pair is multiplied first; for example $2 \times 3 \times 8 = (2 \times 3) \times 8$ and $2 \times (3 \times 8)$.

Benchmark fraction: Common everyday fraction such as $1/2$, $1/4$, $3/4$, $1/10$, which can be used to estimate less common fractions.

Commutative property of addition: Rule which says: The order of the two numbers being added does not affect their sum; for example, $2 + 3 = 5 = 3 + 2$.

Commutative property of multiplication: Rule which says: The order of the two numbers being multiplied does not affect their product; for example, $2 \times 3 = 6 = 3 \times 2$.

Distributive property (distributivity of multiplication over addition): Rule which says: When multiplying the sum of two numbers by a third number, it does not matter whether you first find the sum and then multiply or first multiply each number and then add the two products; for example, $2(3 + 5) = 2(8)$ and $2(3) + 2(5)$.

Exponential function: A function in which the equation ($y = ab^x + c$) contains the variable as the power to which a constant is raised. The graph of this function is curvilinear.

Function: a relationship in which one quantity depends on another. An in-out table is an example of a function.

Linear equation: An equation ($y = ax + b$) whose graph is a straight line.

Linear function: A function whose ordered pairs (x, y), when graphed, form a straight line.

Mean: A number that describes what is average or typical of the distribution, in this case, the arithmetic average obtained by adding up all the items in the data set and dividing by the total number of items; for example, in the data set - 1, 2, 3, 1, 2, 3, 1 - the mean is 1.857 ($13 \div 7$).

Median: A number that describes what is average or typical of the distribution, in this case, the score that divides the distribution into two equal parts so that half the cases are above it and half below it; for example, in the data set - 1, 2, 3, 1, 2, 3, 1 - the median is 2 because there are three items above it and three below it (1, 1, 1, 2, 2, 3, 3).

Mode: A number that describes what is average or typical of the distribution, in this case, the category with the most items; for example, in the data set - 1, 2, 3, 1, 2, 3, 1 - the mode is 1 because there are more 1s than 2s or 3s in the data set.

Order of operations: A set of rules that determines the order for simplifying expressions when more than one operation is involved. PEMDAS is the abbreviation often used to describe the order of operations: P - parentheses (or any type of grouping symbol) E - exponents M - multiplication D - division A - addition S - subtraction.

Pythagorean Theorem: Rule which says: In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the sides. The formula for the Pythagorean Theorem is $a^2 + b^2 = c^2$.

Quadratic function: A function with an equation ($y = ax^2 + bx + c$) that contains squared terms, and two is the highest power to which any term is raised; for example, $3x^2 + 4$. The graph of this function is curvilinear.

Range of data: The difference between the highest and lowest numbers within a set of data.

Relationship: The dependence of one quantity on another.

Root: A factor of a number that when squared gives the number; for example, the square root of 9 is 3.

Slope of a line: represents the steepness of the slant, based on how much it rises or falls for each unit the line moves to the right.

Statement of equality: An equation or expression that shows equality; for example, $2 + 3 = 5$ is a statement of equality.

Statement of inequality: An expression that shows inequality; for example, $2 > 1$ and $3 + 5 < 10$ are both statements of inequality.

Trigonometric function: A function that involves a triangle. Trigonometric functions describe the relationships among the sides and angles of a right triangle. They may also be defined as ratios of coordinates of points on the unit circle.

Y-intercept: The y-coordinate of the point where a graph intercepts the y-axis. The value of y on a graph when x is zero (0).